ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

COMPUTING THE MAKING OF SELJUK GEOMETRIC PATTERNS

M.Sc. THESIS

Begüm HAMZAOĞLU

Department of Informatics

Architectural Design Computing Programme

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ISTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

SELÇUKLU GEOMETRİK DESENLERİNDE YAPIMIN HESAPLANMASI

YÜKSEK LİSANS TEZİ

Begüm HAMZAOĞLU
(523141002)

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Begüm Hamzaoğlu, a **M.Sc** student of **ITU Graduate School of Science Engineering and Technology** student ID **523141002**, successfully defended the thesis entitled **"COMPUTING THE MAKING OF SELJUK GEOMETRIC PATTERNS",** which she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

Thesis Advisor : **Prof. Dr. Mine ÖZKAR** ....
Istanbul Technical University

Jury Members : **Prof. Dr. Gülen ÇAĞDAŞ** ....
Istanbul Technical University

Dr. Elif KENDİR ....
Istanbul Bilgi University

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To my family,

FOREWORD

First and foremost, I would like to express my gratitude to my thesis advisor Prof. Dr. Mine Özkar for her invaluable guidance. This work would not have been achieved without her continuous understanding, and encouragement. Her enlightening insights were hallmarks of ideal academic mentorship. I could not have asked for a more supportive advisor. I am also very grateful to have had the rare opportunity to be a participant in the research project directed by her.

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May 2017

Begüm HAMZAOĞLU
Architect

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ABBREVIATIONS

CAM **:** Computer Aided Manufacturing
CNC **:** Computer Numerical Control
NURBS **:** Non-Uniform Rational B-Spline

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COMPUTING THE MAKING OF SELJUK GEOMETRIC PATTERNS

SUMMARY

This thesis proposes an integrated computational method for analyzing and generating geometric patterns as material things. The study explores the possible contributions of the computational aspects of formal relations between the shapes and their making to the understanding of the design generation in craft and to the contemporary computational design practice and education. The case study comprises a number of geometric patterns carved into stone on monumental building façades in Anatolia from the Seljuk-period. The limited knowledge on the original making processes of these historical designs provides an important opportunity to use an integrated computational approach for understanding the generative process of the stone carving craft as a cultural heritage.

Motivated by the making grammar approach that integrates making in computational design, this thesis investigates its possible applications in analyzing the generative process of making in the particular case of stone carved geometric patterns. The method used in the analysis of the existing patterns relies on analyzing the geometric patterns as material things instead of pure abstract geometries.

The study is comprised of three parts. Firstly, a number of existing stone carved patterns from the Seljuk-period have been analyzed by means of the formal relations between their design layout and the making process. The design layouts of the patterns were examined as compass and ruler constructions that are based on generating regular polygons on circular grids. Various shape transformations were presented in the form of shape rules to indicate the possible generative methods of producing the layouts of the patterns. Seven different types of general transformation rules (tessellation, addition, subtraction, translation, extrusion, rotation, curving) have been highlighted. The layouts of the patterns that can be produced by these transformations are considered as the initial shapes of the carving process.

Secondly, possible scenarios of material transformations for generating the stone carved patterns from the initial shapes were presented in the form of making rules. Making rules developed for stone carved geometric patterns formalize the material transformations that were generated by various making parameters such as tool shape, tool diameter, cut depth and cut distance. The rule-based computational making method presented in this study introduces tool-based emergence as a new concept for the computational making studies.

Finally, the study concludes with examining the application of making rules with today's digital fabrication tools. The application experiments were conducted with a 3-axis CNC milling machine and highlighted the variety of possible pattern generations that were enabled by integrating the knowledge of making parameters

such as tool shape, tool diameter, cut depth and other parameters of the milling toolpath generation.

The results suggest that the rule-based computational making method presented in this study has several practical applications. Firstly, the integrated computational approach for analyzing the generation process of stone carved Seljuk geometric patterns can be used for developing a comprehensive making grammar of stone carved Seljuk geometric patterns. The method used for this study may be applied to other geometric patterns made with various materials, tools, and techniques in future studies.

Formalizing material transformations as making rules has been found useful for reasoning about the formal relations between the surface geometry (curvature, dimension), the rotational symmetry group of the pattern, tools and actions. These findings enhance our understanding of the design generation of stone carved Seljuk geometric patterns.

The design generation experiments conducted with the 3-axis CNC milling machine showed that the rule-based computational making method presented in this study can be useful for enhancing computational design processes by means of diversity and integrity. The results of the experiments from two student workshops suggest that the method for generating carved patterns using making rules can be used for establishing an exploratory making approach that can integrate the craft knowledge, the concept of tool-based emergence and CNC milling technology in today's design education.

Experiments on integrating shape rules and making rules in design processes during the student workshops have been conducted. Although the study is based on a small sample of participants, the findings suggest that the integration of both rule types enhance design generation by means of diversity. Moreover, both shape transformations and material transformations are interrelated and can continuously feed each other.

Lastly, the study suggests a possible contribution to restoration applications by integrating the knowledge of the making of the existing stone-carved patterns. Although the current study is based on a small sample of patterns, the open access database of this research is to be expanded to serve as a base for future collaborations with historians and restorators.

SELÇUKLU GEOMETRİK DESENLERİNDE YAPIMIN HESAPLANMASI

ÖZET

Bu tez çalışmasında, geometrik desenler örneğinde yapım süreçlerinin kural tabanlı hesaplamalı yaklaşımla çözümlenerek hesaplamalı tasarım süreçlerine entegre edilmesi hedeflenmiştir. Çalışma kapsamında geometrik desenlerin tasarım süreçlerini incelemek için geliştirilmiş önceki yaklaşımlardan farklı olarak şekiller ile yapım süreçleri arasındaki biçimsel ilişkileri de hesaba katan ve yapım grameri kuramına dayanan bütünleşik bir yöntem önerilmektedir. Uygulama örneği olarak, Selçuklu döneminde Anadolu'daki anıtsal mimari yapılarda bulunan taş üzerine oyulmuş geometrik desenler ele alınmıştır.

İlk aşamada, geometrik desenlerin yapım süreçlerinin analizi için pergel ve cetvel gibi temel araçlarla seçili desenlerin düzen şemalarının üretimi çözümlenmiştir. Desenlerin üretken sürecinin ilk aşamasını oluşturan düzen şemasının üretimi şekil hesaplaması olarak temsil edilmiştir. Desenlerde görülen çeşitli geometrik biçimlerin pergel ve düzkenar yardımıyla çeşitli çember grid sistemleri üzerinde üretimindeki dönşümler görsel kurallar olarak tanımlanmıştır. Bu kapsamda incelenen desen çözümlemelerinde; döşeme, ekleme, çıkarma, öteleme, uzatma, çevirme ve bükme olmak üzere yedi farklı kurala rastlanmıştır. Bu çözümleme yöntemi ile varolan desenlerin düzen şemalarının biçimsel üretim süreçlerine odaklanarak farklı desen olasılıklarını tasarım sürecine katmak hedeflenmiştir.

Desenlerin geometrik düzenlerinin çözümlemelerinde varolan farklı desenlerin aynı düzen şemalarına sahip olabildiği görülmüştür ve bu farklılıkların uygulanan farklı yapım yöntemlerinden kaynaklanabileceği savunulmuştur. Çözümlenen düzen şemaları kılavuz çizgiler olarak alınarak taş üzerinde oyulmuş desenleri oluşturabilecek yapım yöntemleri yapım kuralları olarak biçimselleştirilmiş ve bu kuralların Selçuklu geometrik desenleri için bir yapım grameri oluşturulmasına katkı sağlaması hedeflenmiştir.

Çalışma kapsamında yapım süreci üç aşamada incelenmiştir. Bu aşamalar; kılavuz çizgilerin yüzeye yerleştirilmesi, oyma işleminin uygulanacağı parçaların veya sınırların tanımlanması ve oyma işlemidir. Bu üç aşamadaki değişkenlere bağlı olduğu gözlenen dönüşümler yapım kurallarını oluşturur. İlk aşamada kılavuz çizgiler, yukarıda görsel kurallarla üretimi incelenmiş olan geometrik kompozisyonlardır. Burada yapım gramerinin başlangıç biçimi olarak ele alınmıştır. Bu çizgilerin farklı yüzeylere yerleştirilmesi ile görülen biçimler dönüşür. Desenler yarım küre biçimindeki kabaralara yerleştirildiğinde biçimlerin eğrilmesi ve uzaması bu duruma örnek gösterilebilir. İkinci aşamada oyma işleminde keskinin izlediği yolun biçim üzerinde nasıl tanımlandığı çözümlenmiştir. Keskinin çizgiyi, yüzey sınırlarını ya da yüzeyi takip ettiği örnekler mevcuttur. Ayrıca çizgilerin paralel olarak öteleme ile çoğaltılarak ya da kalınlaştırılarak yüzeye dönüştürüldüğü örnekler görülmüştür. Son olarak üçüncü aşamada, oyma işlemi değişkenleri mesafe, derinlik, uç kalınlığı ve uç biçimi olarak ele alınmıştır.

Çalışmanın üçüncü ve son aşamasında, sayısal üretim araçları yardımıyla biçim ve yapım kurallarının bütünleşik kullanımının tasarım sürecine olası katkıları araştırılmıştır. Çalışma kapsamında en yaygın sayısal üretim araçlarından biri olan CNC freze makinesi kullanılarak uygulanma süreci temsil edilmiş ve gerçekleştirilen denemeler incelenmiştir. CNC frezeleme yöntemi, taş oyma işlemine benzer olarak çıkartmalı bir üretim türüdür. İşlem, genel olarak makinenin ucuna takılan freze bıçağının ahşap, metal, taş vb plakalar üzerinde dönerek ilerlemesinden oluşur. Bıçağın izlediği yol boyunca bıçağın ucunun kalınlığı, biçimi, adım mesafesi, ilerleme hızı ve ayarlanan derinliğe göre malzeme eksiltilerek şekil alır. Tüm bu işlemler sırasıyla sayısal olarak G-Kod adı verilen üretim kodu halinde bilgisayar programı aracılığıyla makineye okutulur. Günümüzde çeşitli CAM programları sayesinde aracın izleyeceği yolun koordinatları üç boyutlu sayısal modeller üzerinde tanımlanabilmektedir. Daha sonra yine CAM programı arayüzünde kullanılacak araç özellikleri ve kesme işleminin değişkenleri kontrol edilebilir. Bu üretim yönteminde araç uç kalınlığı, biçimi, mesafesi ve derinliği gibi değişkenlerin sayısal olarak kontrol edilebilmesi ve parçaların sayısal modelde tanımlanabilmesi, yukarıda Selçuklu geometrik desenleri için tanımlanmış olan yapım kurallarının uygulanması açısından uygun bulunmuştur. Son olarak biçim kuralları ile yapım kurallarının ortak kullanıldığı ve CNC frezeleme ile üretimlerin gerçekleştirildiği iki adet çalıştay uygulamasının sonuçları sunulmuştur.

Tez çalışmasında sunulan kural tabanlı hesaplamalı yapım yöntemi ile, taş oymacılığındaki keski aracının ucunun biçimi ve boyutundaki değişimler gibi yapım bileşenlerine göre üretilen desenlerin nasıl farklılaştığı ortaya konmuştur. Varolan desenlerin analizlerinde ve yeni desenlerin üretimlerinin denemelerinde aracın özelliklerinden kaynaklanan yapım sonucu önceden tahmin edilemeyen biçimleri oluşması durumu gözlemenmiş ve yapım kuralları ile biçimselleştirilmiştir. Bu bağlamda araç tabanlı belirme kavramı bu tez çalışmasında yeni bir kavram olarak ortaya konmuştur.

Tez çalışmasının katkıları analiz ve üretim olmak üzere iki yönden incelenmiştir. Çalışma kapsamında geliştirilen bütünleşik yöntemin taş üzerine oyulmuş halde var olan Selçuklu geometrik desenlerine ait bir yapım grameri geliştirilmesine katkı sağlayacağı düşünülmüştür.

Yapım süreçlerinde fiziksel nesneler üzerinde gerçekleşen dönüşümlerin yapım kuralları olarak biçimselleştirilmesi, yapım bileşenleri arasındaki ilişkiler üzerine çıkarımlar yapılmasını sağlamıştır. Örneğin; farklı eğriliğe sahip yüzeylere uygulanmış olan Selçuklu geometrik desenleri incelemesinde, yüzey eğriliği, yüzey boyutu, desenin dönel simetri grubu ile uygulamada kullanılmış olabilecek pergel, ip gibi araçlar ve kullanımları arasında ilişki kurulmuştur. Buna göre hesaplamalı yapım uygulamalarında araçların hangi şekilde kullanıldığına bağlı olarak cebirsel ifade edilebilecekleri gösterilmiştir. Taş oymacılığı çoğunlukla birbiri üzerine uygulanan sıralı işlemlerden oluştuğundan yapım süreci hakkında fikir yürütmek zordur. Ancak son üründe görülebilen araç izi kalmışsa bu izler üzerinden fikir yürütülebilmektedir. Bu tez çalışmasında geliştirilen hesaplamalı yaklaşım ile yapım bileşenleri arasındaki ilişkilere dair çıkarım yapabilmenin, taş üzerine oyulmuş Selçuklu geometrik desenlerinin olası yapım süreçlerine dair fikir yürütmeye yardımcı olduğu sonucuna varılmış ve araştırmanın daha çok deseni kapsayacak şekilde genişletilmesi gerektiği düşünülmüştür.

Çalışmanın son aşamasında gerçekleştirilen yeni desen üretimleri, tez kapsamında sunulan bütünleşik ve kural tabanlı hesaplamalı yapım yönteminin hesaplamalı tasarıma çeşitlilik ve bütünlük açısından katkı sağlayacağı düşünülmüştür. Ayrıca üretimler iki öğrenci çalıştayı süresince el ile değil; bir sayısal üretim aracı olan CNC frezeleme makinası ile gerçekleştirilmiştir. Bu sayede, üretimlerin otomasyona indirgenmesi ve hatasızlık kavramları ile eşleştirilen ve keşfe kapalı görülen CNC frezeleme yönteminin tasarım sürecine katılabileceği gösterilmiştir. Üretim denemeleri, hesaplamalı bir zanaat yaklaşımı benimseyerek yaparak tasarlama durumunu ve araç, nesne ve frezeleme işlemi gibi yapım bileşenlerinin özelliklerinden kaynaklanan üretkenliği ortaya koymuştur. Desen çözümlemelerinde olduğu gibi sayısal üretimlerde de araç tabanlı belirme durumlarının ortaya konması, tez çalışmasının en özgün ve önemli katkılarından birini oluşturmaktadır. Buna göre, günümüzde tasarım eğitimi ve pratiğinde giderek yaygınlaşan sayısal üretim araçları ile tasarımcının yapım aşamasında kontrol kazanmasına ek olarak yapım aşamasından beslenebileceğini göstermek hedeflenmiştir.

Son olarak, yapılan çalışmanın olası bir diğer katkısının restorasyon uygulamalarında olabileceği düşünülmüştür. Selçuklu geometrik desenlerinin yalnızca şekilsel olarak değil; aksine yapım süreçlerini de hesaba katarak yeniden üretilmelerinin desenlerin özüne uymayan restorasyon sonuçlarının önüne geçilmesine yardımcı olabileceği öngörülmüştür. Bu çalışmanın sanat tarihi ve restorasyon uzmanları ile birlikte ilerletilebilmesi için daha çok sayıda desenin incelenmesi ve incelenen desenlerin erişime açık veritabanı üzerinden paylaşılabilir hale getirilmesi hedeflenmiştir.

1. INTRODUCTION : MAKING IN COMPUTATIONAL DESIGN

Today, integrating the knowledge of materials, tools and other constituents of making with computational design processes has become more significant with the increasing availability of digital fabrication tools in design practice and education. Designers can take an active role in the production process by controlling the actions of the digital tools through computation. Thus, the integration of design and making processes has reminded the praxis of traditional master builders that acted as both designers and makers in the pre-Renaissance era (Kolarevic, 2005). From then on, the knowledge of making has been optional in design practice and education, and even largely avoided as the intellectual and immaterial activities were regarded as superior to manual skills (Gürsoy, 2016). Mitchell and McCullough (1995) associated this inconsistency with how the knowledge of making and design is exchanged. Accordingly, the distinction of design and making lies in the invention of drawings to extract the design information, whereas the recent integration of design and making is related to the ability to augment drawings through digital modeling and fabrication tools. Hence, as noted by Kolarevic (2003), the future architect might not physically construct a building, but might be able to generate all the necessary information of the construction process.

The impacts of the integration of design and making on computational design has generated the term digital craft in the literature. The term digital craft refers to the crafting of forms by experimenting with material behaviors, fabrication methods and assembly logics (Oxman, 2007). Much of the interest has been on material explorations as many innovative studies suggested new generative design approaches based on the experiments with computable material properties and behaviors (Menges, 2012; Oxman, 2010). These studies suggest a new kind of conceptualization in computational design that is based on digitally informed tectonics similar to the former well-known approach of the Otto and Gaudi's material based form-finding experiments (Oxman, 2012).

In contrast to the material behaviors, very little attention has been paid to integrate the other constituents of making such as tools and motions of actions to the computational design. Often, the digital fabrication tools such as CNC (Computer Numerical Control) machines, 3d printers, laser cutters etc. are used for rapid prototyping purposes. Although the customization of forms is an evident impact on the design process, the knowledge of its making is largely avoided in today's common design practice and education. As such, some designers see the know-how even more unnecessary due to the automation available by the numerically controlled machines. This approach is, in Pye's (1968) terms, a workmanship of certainty. In this approach, there is no exploration of the capabilities of tools and toolpaths and the formation process has no effect in the design ideation process of the product. Yet, the physical effect on the products is inevitable as the material things are results of material formation processes that are done with particular tools in particular ways. In fact, as Schodek et al. (2005) put it, the numerical controls do not replace the knowledge of making, instead, the user needs to be fully familiar with the methodology of the machine in order to produce an intended outcome.

In general, the responsibility to be familiar with the digital fabrication tools is often given to manufacturers and avoided by the designers. Yet, some researchers have focused on fabrication-informed design processes to promote innovative design generations. Each fabrication method has its own advantages and limitations by means of its particular action (adding, subtracting, milling, burning, bending etc.) and movement capabilities. Typically, the method is chosen according to the desired shape and material for prototyping purposes. On the other hand, the unusual spaces and surface geometries of the 1:1 scale projects constructed with these tools in educational areas show that the digital fabrication tools enable new form-making methodologies (Iwamoto, 2009). Accordingly, for instance, laser cutters enable the generation of sectioned and sequenced forms and structures, such as waffle structures, as they produce sheet materials, whereas CNC routers are used for contouring of solid surfaces or molding and casting of fluid materials in order to create three-dimensional forms. Iwamoto (2009) also shows a number of projects that have examined the customization of toolpath as a form-finding tool. For example, the project named Tool-Hide by Ruy Klein articulates an iguana skin-like surface texture by developing a particular toolpath that transforms a digital circular pattern drawing into scallops (Figure 1.1). In another project named Satin Sheet by Heather Roberge, a pattern drawing that is the repetition of a single shape has been materialized and transformed into a homogenous unified surface with varying depths by generating an algorithmic force-field based digital model that could then be produced by CNC milling (Figure 1.2). The last example does not include a full control and customization of the CNC toolpath, but the impact of the tool's movement capability is evident on design generation. All in all, these examples indicate the notion of workmanship of risk (Pye, 1968). The workmanship of risk refers to the explorative kind of making that allows mistakes but at the same time may result in emergent outcomes. Pye (1968) describes the difference between the workmanship of certainty and the workmanship of risk by comparing printing with writing with a pen. Accordingly, printing indicates a more controlled process, certainty, and perfection by means of accuracy, whereas writing with a pen involves risk but also the ambiguous nature of exploration by interaction. In a recent study, Gürsoy (2016) refers to Pye's differentiation by examining the impact of making in design ideation process by calling one making of and the other making for. Thus, making of indicates the making of things and the workmanship of control, whereas making for indicates the exploratory making of workmanship of risk.

Moreover, there has been little discussion about the relation between the tools and the geometry of the becoming forms so far. One of the studies that highlight this relation presents the generation of three-dimensional ruled surface geometries using the hot-wire cutting method (Kieferle et al., 2008). Thus, paying attention to the relation between the geometry and the tool's movement capabilities resulted in variable form generations that enhanced the design ideation process (Figure 1.3).

The contribution of digital tools to the computational design process is not only form-finding methods but also performative outcomes. For instance, a load of opaque materials like cardboard sheets may become a transparent surface by sectioning and layering (Figure 1.4). In another major study, Oxman (2007) presents experiments on form generations that couple the production and assembly methods with material behaviors and therefore enhanced the computational design process by augmenting shapes with both the material and movement information. Oxman's experiments use variable digital making methods such as cutting, scoring, etching and stretching together with physical actions such as stretching and folding in order to generate variable forms through flexible material behaviors. Thus, integrating material behaviors with movements in the making process present a unified computational design approach that can enhance both the creativity and practicality in the usual practice. Menges & Schwinn (2012) compares this integrated and performative approach to the biological systems by means of relation and adaptation of form, structure, and performance.

Figure 1.1 : CNC milled patterns from the project "Tool-Hide" by Ruy Klein (URL 1).

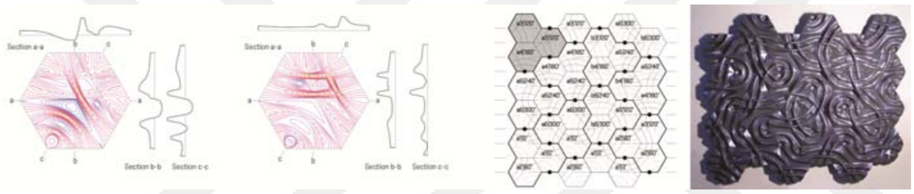

Figure 1.2 : CNC milled patterns from the project "Satin Sheet" by Heather Roberge (Iwamoto, 2009).

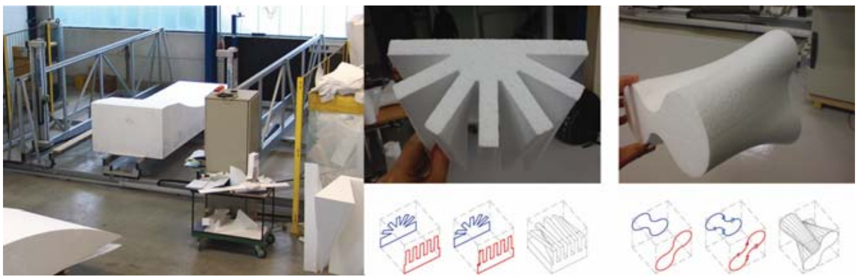

Figure 1.3 : Hot-wire cut models of ruled surfaces (Kieferle et al., 2008)

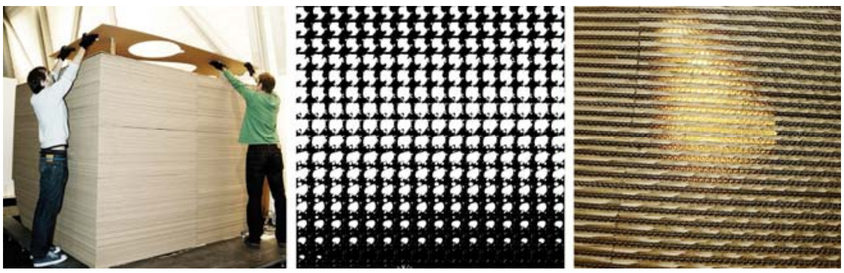

Figure 1.4 : Translucent surface of the layered laser-cut cardboards from the project "Mafoombey" byMartti Kalliala, Esa Ruskeepää, and Martin Lukasczyk (Iwamoto, 2009).

The integration of the knowledge of making in computational design is also strongly related to how the knowledge is computed in the process. Throughout the years, there have been several attempts that put material and fabrication information together in scripted algorithmic codes, but only a few focused on integrating the information in the design generation process. Sass and Oxman (2006) presented a model for producing design variations at the small-scale component level by integrating rapid prototyping limitations to the generative computing. Their work is focused on the assembly details. Similarly, in another recent study by Sachs (2015), a G-code scripting algorithm has been developed for producing a pre-designed folded triangulated shape with variable material properties (strength, thickness etc.) with computed manufacturing information such as connector elements (holes, folds etc.) and assembly data. Even though Sachs' work does not put the fabrication and material information in the early stage of the computational design process, the result provides insight to the formation process at a detail level. Sass and Oxman (2005) addressed the issue, that integrating rapid prototyping in generative computing may transform the design into a paperless process. Hence, the metaphor of being paperless recalls the notion of the traditional practice of master builders as mentioned earlier. Moreover, a recent study by Loh et al. (2016) argued that the integrated process of digital craft process needs a more dynamic workflow to achieve the creative process of craft. Their suggestion is that custom scripting of machine codes will enable a more integrated workflow of computation in order to include the interactive and dynamic notion of craft. Similarly, El-Zanfaly (2015) proposes a do-it-yourself approach in design making, that encourages designers to start with learning how the tools work in order to gain full control of the making process. This approach is becoming more relevant as today's open source culture is making personal fabrication more available (Gerschenfeld, 2005). However, the computational aspects of the generative process of making and its relation to design thinking are yet to be discovered.

The dynamics of the generative computation is as significant as the development of digital production methods in order to integrate the knowledge of making to the computational design process. In fact, the motive of computational design is about the description of transformations and interactions in shape formations with rules (Aranda & Lasch, 2006). Therefore, as Kendir and Schork (2009) points out, the transparency in the design ideation process allows a more extensive craft approach in computational design that comprises both analog and digital tools.

Shape grammars, introduced by Stiny and Gips (1972), have been used and developed extensively by many researchers for analyzing and applying design generations in computational design. The significance of computing with shapes lies in the integration of visual thinking and reasoning in computation. Thus, shapes are formal representations of what we see and therefore they possess infinite parts that enable computing emergent shapes with infinite part relations (Stiny, 2006). Visual computation of shapes based on formal algebras provide an explorative way of design thinking in computational design. The methodology is based on describing and specifying the transformations in the form of shape rules. This approach has been used for decoding and describing design styles as a process rather than a finished product (Knight, 1994). Özkar and Lefford (2006) argued that, based on their examinations of the generation process of Seljuk and Celtic patterns, the transformations of shapes are related to the materials, tools and all other components of the making process and therefore the artist's generative process can only be understood by integrating all these components. A recent study by Gürsoy and Özkar (2015) developed a framework for formalizing the making process with shape rules (Figure 1.5). Their method formalized the physical manipulations on dukta models by hands and the resulting shapes. The result indicates an analytical study of the physical transformations by abstracting the point of the manipulations with labels. This approach promotes visual reasoning of making in design practice and education (Gürsoy, 2016). Recently several researchers have examined the formalization of making using shape rules. For instance, Harrison et al. (2015) formalized folding of paper models and the resulting shapes by shape rules (Figure 1.6). The formalism changes as different actions require different labels and other properties such as weight formalisms. Thus, Harrison et al. (2015) used different lines such as continuous, dashed or dotted lines for formalizing the actions. In another study by Gürsoy et al. (2015) sensory aspects of material manipulations have been formalized in form of shape rules. The study suggests visual reasoning of material manipulations and behaviors by examining it on the variable light patterns on perforated cardboard material. Another recent research by Noel (2015) illustrates the traditional wire-bending craft in the form of shape rules (Figure 1.7). The shape rules of the wire-bending grammar abstract the craftsmen's actions at a connection detail level and use shape formalisms with varying color, thickness, size and texture for describing different materials such as wire, aluminum rods, fiberglass tape, cable tie etc. Hence, the wires are formalized as basic lines, it allows the computation of wire-shapes visually trackable. Yet, the formalism of the other materials and tools lack this feature.

Moreover, the studies presented thus far on formalizing making with shape rules were focused on analog processes. Yet, several other experiments have been conducted for formalizing digital production processes with shape rules. For instance, Jowers and MacLahlan (2014) formalized the material behaviors on 3d printed multi-material surfaces by defining weight functions in addition to the shapes. Another recent study by Bidgoli and Cardoso-Llach (2015) focused on the motion of the tools and proposed a motion-grammar for robotic hot wire-cutting. Their work abstracts the wire-cut foam surfaces as ruled surface geometries using NURBS (Non-Uniform Rational B-Spline) definition in the digital medium. In that formal description, the ruled surfaces are defined through surface segments with their curvilinear boundaries, that changes its shape according to the NURBS degree in the curve definition (Figure 1.8). Hence, this formalism specifies an unusual part relation of shapes that presents a well-adjusted spatial relation to their physical manipulation by robotic wire-cutting. Thus, the formalism enabled the authors to reason about the relation between the geometry and the sequence of the motions since, for instance, the robotic arm needs to finish one part before starting another if there is any intersection.

All in all, the experiments on formalizing making with shape rules present a generative computational framework to be used in design practice and education. Integrating shape computation and making suggests a more transparent and generative making process that reveal possibilities and limitations of tools, materials and their relations with a range of becoming forms. Moreover, the variety of formalization studies provides some of the initial aspects of the computational making.

One of the important aspects of formalizing computational making is the scale issue. Although the shapes are the abstract formalization of what we see on material things and therefore are not related to physical quantities such as scale or dimension, the formalizations need to differentiate for describing the actions at different scales. For instance, in Noel's (2015) work, the shape computations of the generation of a wire-bent half-dome is formalized by illustrating only wires. Yet, the other materials in the associated shape rules disappear. In that sense, the descriptions of the craft actions and the descriptions of craft computations do not fuse together. Therefore, the algebraic operations do not work. On the other hand, Jowers and MacLahlan's (2014) method suggests augmenting shapes with weight functions in order to allow algebraic computation of the variable formal surface generations with material features such as flexibility, hardness and mixture value of the composites, that are either numerically or vectorially described. This semi-formal approach on formal description presents an explorative framework for using additive manufacturing, yet this method might get very complex and difficult to use when dealing with making processes with much more parameters.

Moreover, another important aspect of formalizing computational making is the dynamics of the process by means of sequencing and complexity. The fact in most cases the transformations in a making process has a progressive nature. Therefore, the formal relations between tools, materials and becoming forms might not be visually tractable when transformations are described as shape rules that go from one step to the next in a finite range of sequences. For instance, Bidgoli and Cardoso-Llach's (2015) formalized the motion rules as descriptions of transformations from one surface geometry to another as a result of many parameters in the process that are defined through verbal labels attached to the shapes. These verbal labels indicate the motions. Yet, the formal relations between the shape and each motion is hard to see on the overall metamorphosis of the surface geometry. Therefore, there is a need for a more closer examination of the formal impact of each type of tools and actions in order to associate the generative process of making with the formal aspects of the computation. Describing the formal relations between each type of tools and actions is crucial for computing any kind of making process since their effect on the part relations is significant.In this context, it is important to consider that shapes are formal descriptions of what we see and therefore they are not associated with any kind of physical features. Therefore the usual way of shape computation with algebraic part relations and reversibility of abstract notions do not work when computing with material things. The part relations in shape computation relies on that the lines fuse (Stiny, 2006), yet the materials often don't. Therefore, formal descriptions of material things require specifications of formal relations. Moreover, in addition to the physical reversibility of the actions, the sequencing which is the essence of the workflow of material practices, as mentioned by Loh et al. (2016), needs to be considered for integrating the constitutive features of making in design computation.

Recently, a new approach that extends shape grammars into making grammars have been proposed by Knight and Stiny (2015). The making grammar establishes a new theoretical framework for computational design that considers designing a kind of making and extends the shape studies into a study of material things. Therefore, they suggest a more integrated framework that augments shapes with other material descriptions in form of making rules. The significance of this method lies in the extensive formalization of the actions and material behaviors in order to reveal the formal relations in between. The proposed formalization of making rules is expected to examine and decode all constituents of the making process such as things and the activities in the process (Figure 1.9). Accordingly, things are materials and tools, whereas the activities are any kind of doings and sensings. Knight and Stiny's work highlights the computational aspects of making by means of the algebra of making grammars, part relations (such as embedding of strings in the knotting grammar) and time-dependency, i.e. sequencing. They suggest that algebras and related part relations need to be reconsidered for each operation together with the time parameter that can be integrated into the computation through labels. Later, Knight (2015) examined the notion of algebras for stuff in the case of knotting grammars and argued that, algebras for stuff are significant to reveal how the manipulations in making by means of sensory and experiential features. Therefore, the applications of making grammars in different cases are promising and yet to be discovered.

Motivated by the making grammar approach that integrates making in a significantly comprehensive way in computational design, this thesis aims to investigate its application in computational design for analyzing the generative process of making in the particular case of geometric patterns carved into stone on monumental building façades in Anatolia. The thesis study starts with analyzing the making rules of the analog methods of medieval stone carving from Seljuk-era and concludes with examining the application of making rules with today's digital fabrication tools.

Thus, a method for analyzing and generating geometric patterns as material things has been presented.

The main motivations of this thesis study are three-fold. The first one is to explore the possibilities of the making grammar approach by means of analyzing and generating geometric patterns as material things. The case study focuses on the making of stone carved Seljuk geometric patterns. Seljuk geometric patterns are examples of Islamic geometric patterns that were used as ornaments on monumental building façades in 12th-century Anatolia. Various shapes can be seen on these patterns, such as polygons, stars, and lines. Geometrical constructions of these shapes have been subject to many studies. Yet, there are only a few studies that focus on the making process of these patterns. Seljuk geometric patterns are material things that are made of stone, wood, brick or ceramic tiles using special craft techniques. Materials, tools and all other components of the making process such as hand movements are all related to the resulting shape of the pattern. Yet, there is very little information about the making of Seljuk geometric patterns today. The knowledge of the craft techniques from the Seljuk-era is not stored in written resources since these techniques were carried by words of mouth and stored in the memories of the masters (Mülayim, 1982). Therefore, the analysis of pattern generations should rely on clues on the existing patterns. However, it is hardly possible to track the whole process on the final product, since stone carving is a progressive process, that consist of a series of different subtractive actions (Wootton et al., 2013). The construction lines mostly vanish during the making process. There have been two major valuable studies that examined and discussed the craft practice of stone carving in Anatolia. One of which examines the historical findings from the medieval era and traces the construction lines on the existing patterns (McClary, 2017), and the other examines the impact of on-site material and social interactions in the craft practice of stonemasons in today’s Anatolia (Kendir, 2014).

The limited knowledge of the making of stone carved geometric patterns provided an important opportunity to examine the use of the integrated computational approach for understanding the generative process in the cultural heritage of craft. Thus, a possible contribution to this field formed the second main motivation of this thesis study.

The final experiments of the study examine the application of making rules with digital fabrication tools. The method is simply the translation of the making rules that were derived and learned from the analog tools into the digital tools. 3-Axis CNC milling tool has been chosen for its similarity to the subtractive production method of stone carving. The experiments were conducted with a group of student volunteers, where they learned the making process of CNC milling and used the transformative rule for generating new designs. In that context, the third main motivation of the study is to discuss possible contributions of making grammar formalism for establishing an exploratory making approach in design education.

The rest of the study is comprised of four parts. Chapter 2 provides the reader the necessary background information related to the making of geometric patterns in medieval Anatolia. Chapter 3 constructs a detailed study on analyzing the formal relations between the design layout of existing patterns and their making process. The outcomes of this chapter provide the possible geometric guidelines that can be used as the initial shape in the making process. Chapter 4 presents the making rule formalizations and the represent the computational aspects of the making of Seljuk geometric patterns. Finally, Chapter 5 presents the experiments with the CNC milling machine.

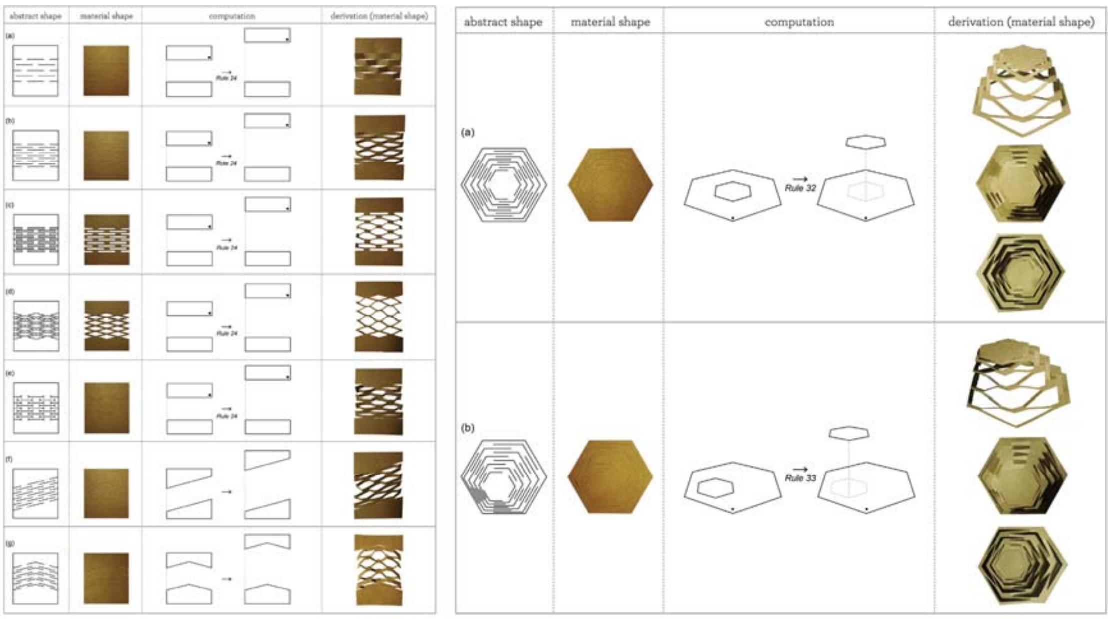

Figure 1.5 : Visualisation of material manipulations by shape rules (Gürsoy & Özkar, 2015).

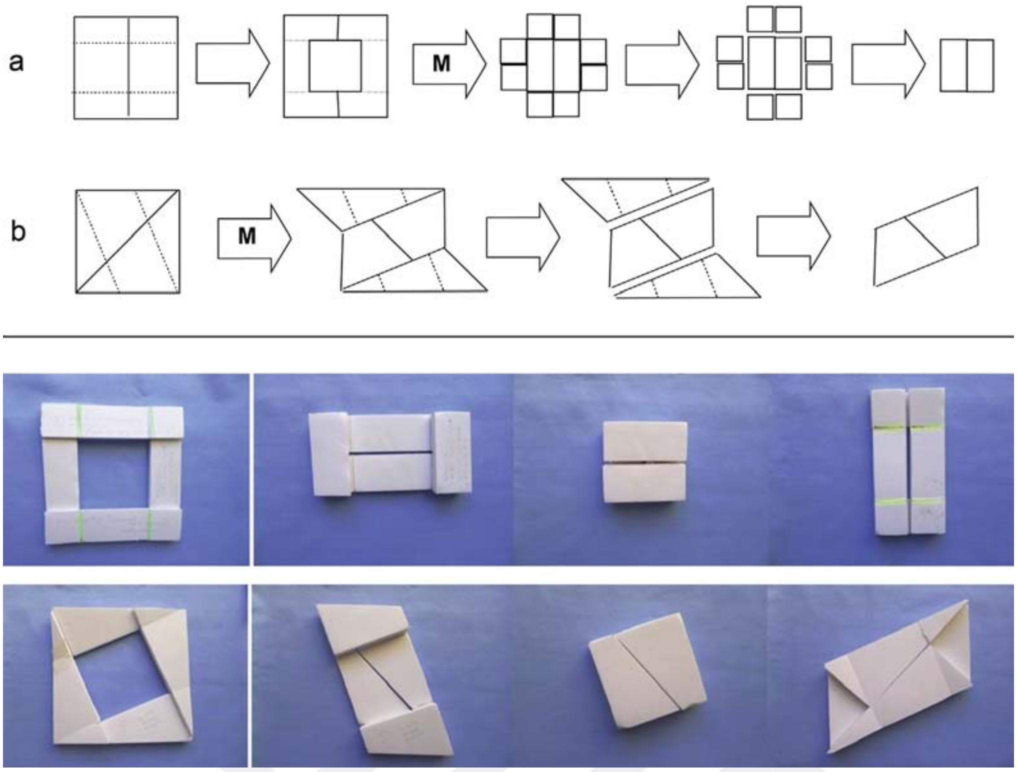

Figure 1.6 : An experimental shape rule schema for selectively removing material and the resulting model (Harrison et al., 2015)

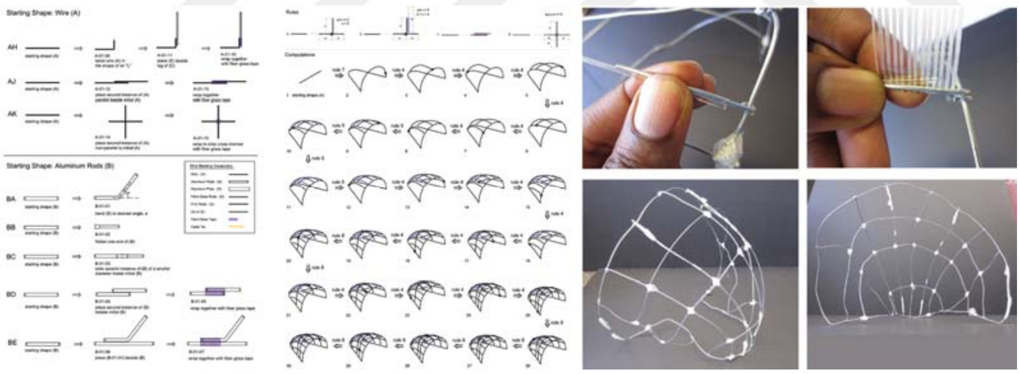

Figure 1.7 : (From left to right) Sample wire-bending rules, generation of a wire-bent half-dome with the rules and the hands-on application (Noel, 2016).

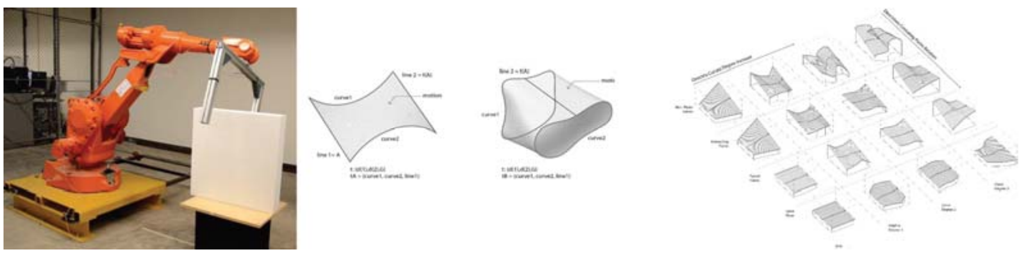

Figure 1.8 : (From left to right) Wire-cutting tool, example motion rule and sample vocabulary of motions (Bidgoli & Cardoso-Llasch, 2015).

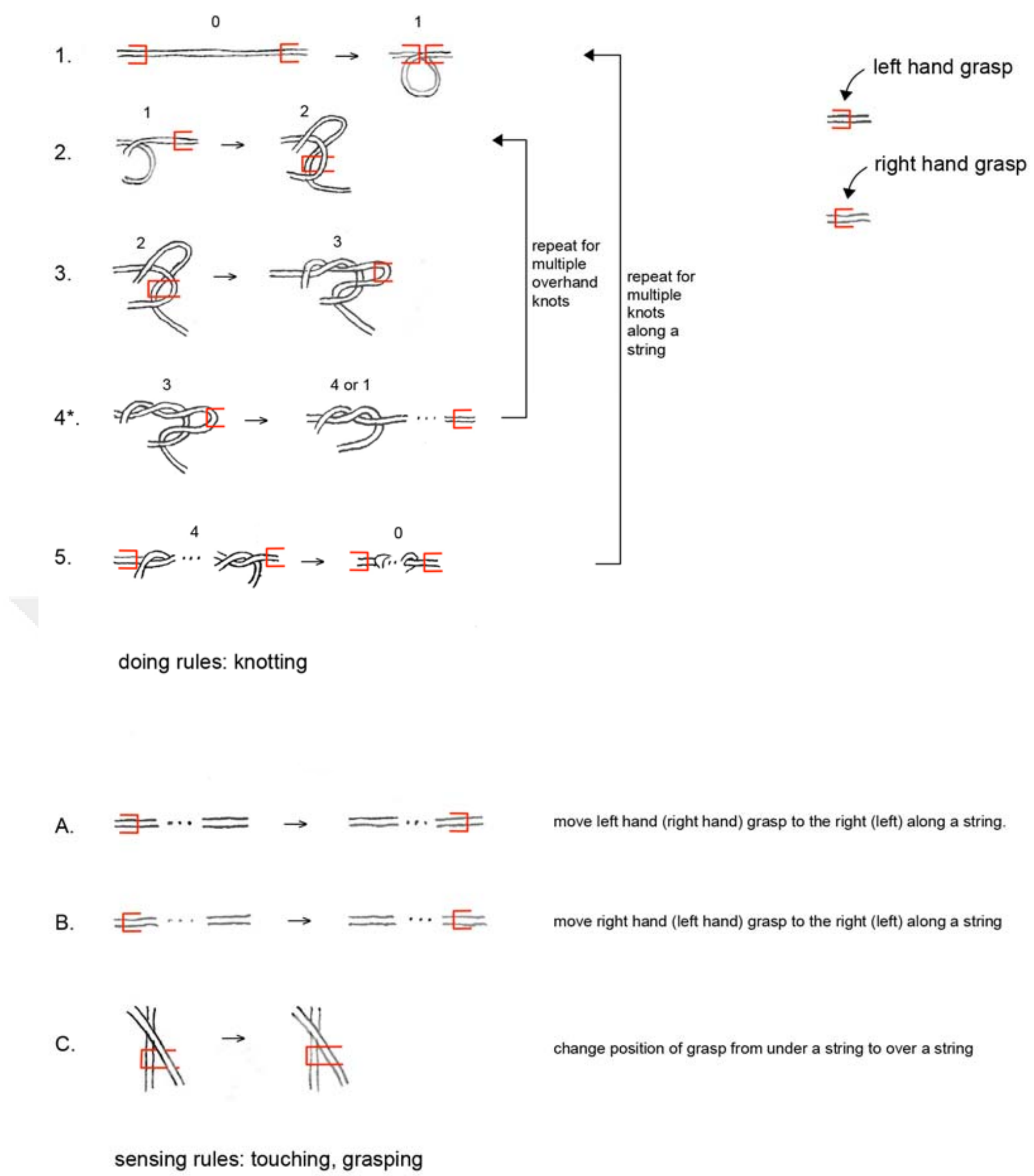

Figure 1.9 : Sample making rules for computing with knotted strings (Knight & Stiny, 2015).

2. MAKING OF GEOMETRIC PATTERNS IN MEDIEVAL ANATOLIA

For many years, the geometrical construction methods of the ornamental patterns from the Seljuk period of Anatolia and other adjacent Islamic cultures have been subject to many interdisciplinary studies. Since there are only a few written original sources that address the historical techniques from that era, many researchers developed unique methods for analyzing the generation of these complex geometrical shapes. Hankin (1925), Critchlow (1976) and Bakırer (1981) proposed that the geometrical compositions of the patterns can be constructed by drawing with compass and straightedge. Hankin's (1925) method is based on dividing the surface into polygonal tiles that are in contact with each other. The tiling-based approach has been further studied by Kaplan (2000; 2005) in order to derive patterns from computer algorithms. Kaplan (2000) also developed a Java applet to produce patterns with the same principle. Furthermore, a considerable amount of literature has been published on the generation methods of quasi-periodic types of the patterns. Al Ajlouni's (2011) model differentiates from the traditional methods based on local rules and suggests a global long-range order based method that can be applied by using compass and straightedge. In another well-known study by Lu and Steinhardt (2007), a set-based method of girih tile tessellation for constructing the quasi-periodic patterns was introduced. The quasi-periodicity is often found on patterns with 5-fold and 10-fold symmetries that more likely consist of pentagons and decagons. Hence, Cromwell (2009) discusses that it is possible that the artisans in that period were aware of the problem of repeating patterns with 5-fold and 10-fold rotational symmetries on a plane and therefore generate these quasi-periodic patterns by transformations such as reflection, rotation, and translation of the existing geometries. Cromwell (2009) concludes that the fact that artisans were able to produce quasi-periodic geometries does not mean that they know the concept of quasi-periodicity. This approach draws attention to that examining the possibilities of the primitive tools are more reliable than the new conceptual inventions since the design ideation process from that era is mostly unknown. In another major study, Jowers et at. (2010) highlights that emergence in the compass and ruler construction method occur in the process and therefore is related to the design space, whereas emergence in the set-based method occur as a product and is not related to the design space. Thus, Jowers et. al.'s (2010) study supports the idea that set based and motif based approaches may be limited the describe generation processes of some variations.

A number of innovative studies have examined the use of computational design methodology for analyzing the design generation of Islamic geometric patterns. Studies show that a shape grammar for Islamic geometric patterns can be developed by examining the part-whole relations on the patterns (Cenani ve Çağdaş, 2006; Ulu, 2009). In a more recent study, Özkar (2014) suggests an integrated computational framework that relates the tool and material knowledge of the Seljuk-era to the formal relations in the generative process of the artisan.

The construction techniques from the Seljuk period have been subject to few but valuable studies. Özdural (2000) introduced and discussed the regular meetings between geometers and artisans in the medieval Islamic world, where the geometric methods necessary for applications were demonstrated by geometers visually. Özdural's (2000) study is based on written mathematical sources, one of which is Abū al-Wafā' Būzjānī's (ca. 940–998) scripts on geometric constructions addressing the artisans. Sarhangi (2008) highlights that the interactions between mathematicians and artisans can be seen on the spherical tessellation drawings of Al-Būzjānī, that shows the three-dimensional constructions with two-dimensional representations. Thus, Sarhangi (2008) concludes that the mathematician was relying on the 3d visualizations in craftsmen's minds. In that way, the complex construction methods may have become more comprehensible for craftsmen to apply. In another major study, Necipoğlu (1995) reported that the introduction of Al-Būzjānī's manual comprises basic geometric constructions such as constructing regular polygons inscribed in circles. For instance, Figure 2.1 provides some of the drawings from the manual that show various regular polygons such as triangle, square, pentagon, hexagon, heptagon and dodecagon inscribed in circles. The marks on the corners of the polygons that intersect the circles indicate that the polygons were generated by subdividing the circles. Similarly, other studies have reported the construction methods of muqarnas units and layouts based on the writings of the 15th-century mathematician al-Kāshī (Dold-Samplonius, 1992; Koliji, 2012).

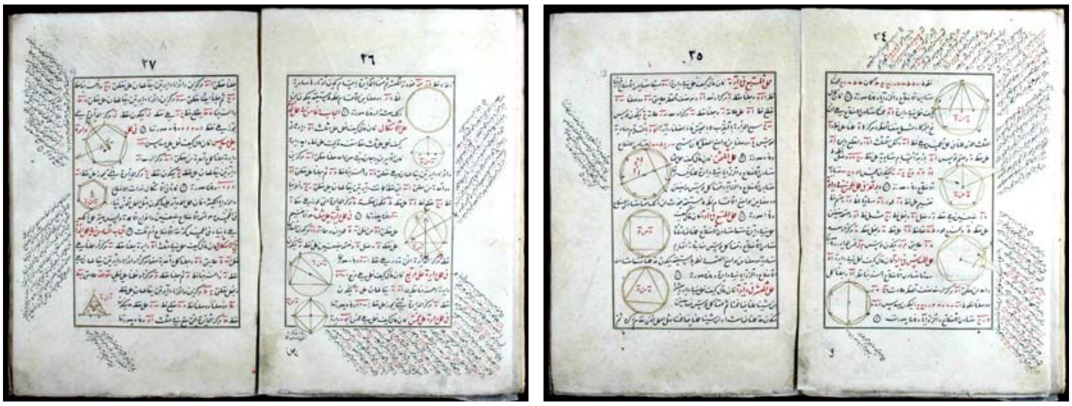

Figure 2.1 : Drawings from Abū al-Wafā' Būzjānī's manual showing various regular polygons inscribed in circles (Būzjānī', 10th century).

In view of all that has been mentioned so far the design generation of patterns were largely examined as abstract geometries regardless of how they were affected by material properties, tools and formation methods that the craftsmen interact with. The only exception is the consideration of compass and ruler method for generating the layouts. Though, investigating the patterns as material things is beneficial to obtain information from the tool marks on the existing patterns. Thus, some scholars rely on such clues to speculate on how they were made. Seljuk geometric patterns were applied to different materials such as wood, stone, brick, and ceramic. The geometrical compositions differentiate inevitably based on the potential techniques that can be done with specific materials. Bakırer (1981) examines the making of brickworks as part of the formation of the geometry, instead of an automated construction of a predetermined shape. Her drawings of patterns examples from medieval Anatolia demonstrate the relation between the assembly of the bricks and the emergent geometric outcomes. In a more recent study focusing on the patterns from Persian architecture, Kharazmi and Sarhangi (2015) compared two methods of ornamental brickwork construction, which are using standard sized bricks and shaped bricks. Their argument that producing customized shaped bricks were useful for generating complex motifs, is an interesting example of reasoning about the relations between the material knowledge and design generation. Moreover, a large part of their study focuses on the stucco works from the same period, where they analyze the layout of the existing stucco patterns. This study is one of the few examples that draw and analyze the formal relations on the images of existing patterns. Yet, their approach does not consider the dimensions of the material such as the thickness of the lines but only focus on the abstract layout of the patterns. Therefore, the relation between the making of stucco patterns and their geometries is not presented. Similarly, the making of geometric patterns from ceramic tiles has been pointed out in a recent study by O'Kane (2016), where he examines the placement marks and joints on the tilings.

Much of the available literature on the making of Islamic geometric patterns comprise the regions outside of Anatolia. Moreover, even fewer literature is available on the stone carving methodology in general. A recent major study by McClary (2017) presents a comprehensive research on the tools and methods of the craftsmen and their working conditions in Anatolia from circa 1170 to 1220. His studies report two distinct aspects of the methodology of stone carved ornaments. The first one is that the stone was a new material to the Seljuks but the familiarity of the local craftsmen with stoneworking techniques leads to the generation of a new and unique aesthetic in Anatolia based on the application of the Seljuk-style to an unusual material. Based on McClary's report, some particular features of stone carved patterns may be interpreted as the material based differentiations. For instance, the smooth transitions, rich variations of depth and curved profiles may be the emergent results of this encounter with the stoneworking techniques. The second aspect mentioned by McClary is that the tool marks on the finished stone artifacts provide information about the type of tools and methodology from the Seljuk-era, but such an analysis requires an exceptional close examination of the surviving marks. Thus, McClary points out the existing construction lines on the patterns that may have been used as guidelines for the construction. These findings support the view of another significant study by Bakırer (1981), that presents compass and circle traces on a stone piece in Divriği Great Mosque in Sivas. Figure 2.2 shows the image of the tool marks, the representative grid of interlacing circles drawn by compass and an exemplary resulting pattern layout from the same monument. Bakırer's findings demonstrate the use of circular grids for applying the geometric patterns on the materials. The evidence of the use of inscribed circular grids on construction area gives new meanings to the formal computations of the compass-ruler method by integrating the material and surface specifications. Özkar (2014) highlighted the possible benefits of using circular grids for applying complex geometric patterns on uneven surfaces. Hence, complex design symmetries can be calculated and preserved by using primitive tools and techniques.

Overall, these studies highlight the need for a more integrated approach to the understanding the making of Seljuk geometric patterns. The examinations of more examples of existing patterns may provide insights to the research in this context.

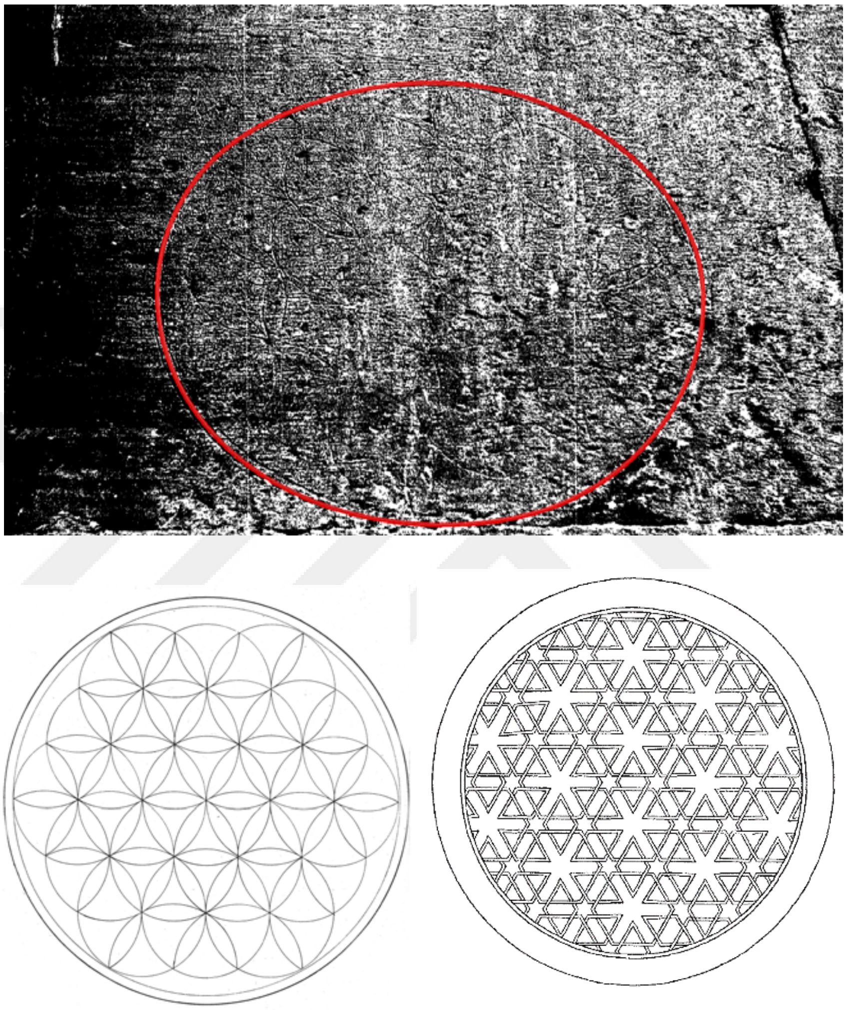

Figure 2.2 : The tool marks (first row), the representative grid of interlacing circles drawn by compass (second row left) and an exemplary resulting pattern layout (second row right) in Divriği (Bakırer, 1981).

3. ANALYSIS OF THE SHAPE GENERATION PROCESS

Stone carved geometric patterns from the Seljuk-era appear as uniform solid surfaces carved on discrete stone blocks. There are examples of patterns that were applied on a single block, such as the pattern on the engaged column at the entrance of the Tomb of Mama Hatun near Erzincan (Figure 3.1-a). On the other hand, there are patterns that were placed on a series of blocks, such as the pattern on the façade of the same monument (Figure 3.1-b). It is hardly possible to conclude whether the patterns were carved before or after the assembly of the blocks. The repetition frequency of the patterns is mostly identical with the dimension of the stone block. In other words, the pattern on each block is the same and the sequence of the blocks is unimportant. Yet, there are examples where the pattern is not equally distributed on multiple blocks, such as the pattern on the engaged column at the entrance of the hospital in Amasya (Figure 3.1-c). This distinction might indicate that the patterns were carved after the assembly of the wall. On the other hand, the patterns may have been remade or reassembled due to possible damages throughout the years.

(a) (b) (c)

Figure 3.1 : Examples of different assemblies of geometric patterns on stone blocks.

In any case, the surfaces consist of multiple levels of height. The variation of the levels creates a contrast between different parts of the surfaces. Therefore, discrete parts can be identified by the boundaries of the parts on different levels. These boundaries can also be the guidelines that were used in the carving process. The guidelines are the geometrical shapes that the stone carved patterns are generated from. However, there are different possibilities. The carving tool can be used exactly on the guidelines or along them with a particular distance on the left or right side. If the guidelines are closed shapes, then the direction of the tool can be described as inside or outside of the shape. The toolpath depends on the decision of the craftsmen.

The guidelines of the patterns should be geometrically constructible using basic tools like compass and straightedge. This study relies on the circular grid method that was introduced by Bakırer (1981) for constructing the geometric shapes. Therefore, the whole process of the shape generation from a single circle to the overall geometric composition was examined for each pattern. Thus, the symmetry of the patterns was matched with the circular grid structure.

In order to analyze the guidelines on a pattern, various possibilities should be drawn and examined. The first example is the pattern on the engaged column at the entrance of the hospital in Amasya (Figure 3.2).

Figure 3.2 : Photos of Pattern I which is on the engaged column at the entrance of the hospital in Amasya.

Firstly, the pattern wraps around the cylindrical surface and has a six-fold rotational symmetry. Therefore, the shape generation is expected to start from drawing a six-fold circular grid. The process of constructing the grid starts with drawing one circle using a compass. Then, another circle can be drawn by placing the steady arm of the compass on any point on the first circle and opening the arms until the rotating arm reaches to the center of the first circle. The result of this drawing will be two circles with the same radius intersecting each other and having the central point on the perimeter of the other. This particular shape is called Vesica piscis in Euclid's Elements. After that, the third circle of the grid can be drawn by centering it on the intersection of the first two circles and using the centers of these two circles as reference points for determining the rotation path of the compass and therefore the radius. The other circles can be drawn by using the center point of the existing circles and their intersection points until there are six identical circles that interlace each other around the first circle with a six-fold symmetry. Figure 3.3 shows the process as a series of shape computations in order to illustrate how the circular tessellation of the shapes can be generated by using only basic tools and visual computation. The first shape is labeled with a plus sign in order to track the transformations visually. Moreover, two more labels were used in this illustration. The x marks indicate the center of the newly added circle in each step and therefore the location of the steady arm of the compass and the circular marks indicate the reference points that the compass was rotated through.

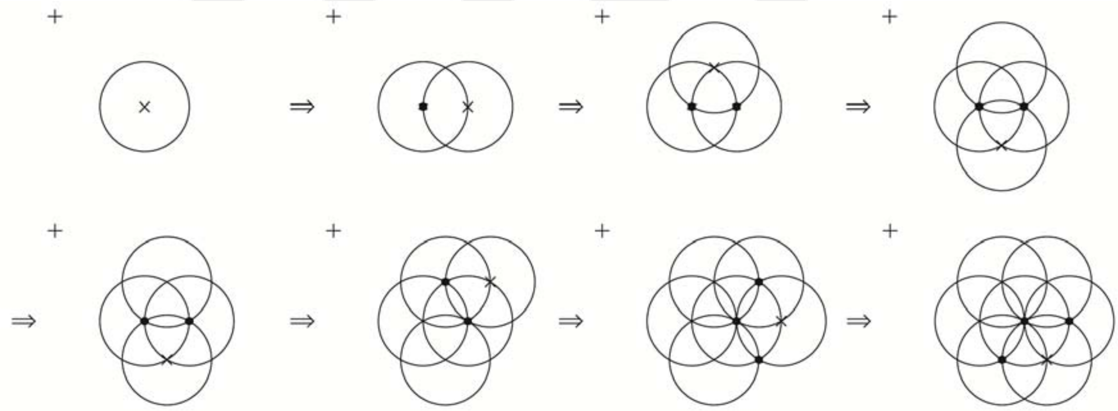

Figure 3.3 : Sequential shape computations for generating a grid of interlacing circles with a six-fold symmetry.

Secondly, the geometrical composition is drawn on the circular grid. In order to analyze from which geometrical composition the existing pattern can be generated, different possibilities were drawn based on the three discrete parts on different levels of the pattern. Moreover, the geometrical composition should be geometrically constructible on the previously drawn circular grid and preserve the six-fold rotational symmetry. Therefore, firstly, the circular grid is placed based on the repeating shapes on the digital image of the pattern and then, three different possible guidelines were tessellated on the circular grid. The first one goes through the central axes of the repeating parts at the highest level, whereas the second one goes through the inner boundaries and the last one goes through the outer boundaries of the same parts (Figure 3.4). The drawings show that the first and the second geometrical composition fitted the six-fold symmetry, whereas the second one didn't fit and formed new shapes that do not belong to the original pattern. The pattern might have been generated from both compositions. On the other hand, there is another pattern on the same monument, that appears to have a similar underlying geometrical composition. Therefore, the same analysis of possible guidelines was also done for the second pattern in order to find out if both patterns can be generated from the same geometrical composition.

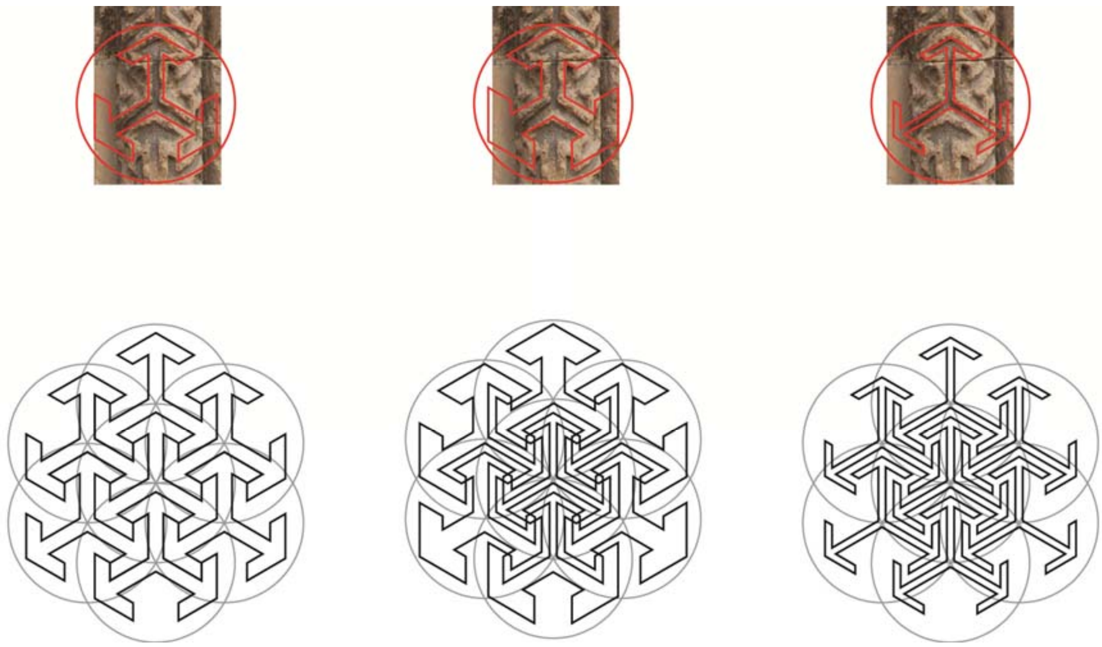

Figure 3.4 : From left to right: Drawings of the lines that go through the central axis, outer boundaries, and inner boundaries of the repeating shapes.

The second pattern example is situated very close to the first pattern at the entrance of the same monument (Figure 3.5). On this example, the pattern is placed on a flat surface and the color of the stone blocks is different than the previous example. The patterns appear to have two main discrete repeating parts that are separated by engraved lines. In this case, three different possible guidelines were drawn based on these two discrete parts and tessellated according to the circular grid. The first one goes through the central axes of the thin parts that surround the thicker parts. The second one goes through the inner boundary of the thin parts and the last one goes through the central axes of the thicker parts (Figure 3.6). The first drawing is identical with the first drawing of the previous pattern and therefore can be used for generating different patterns.

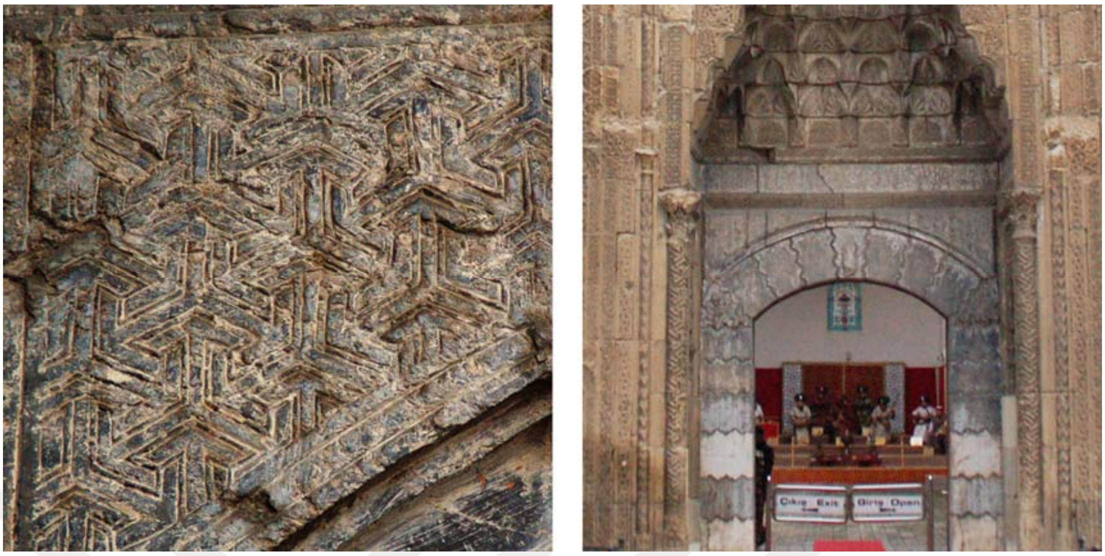

Figure 3.5 : Photos of Pattern II on the same monument.

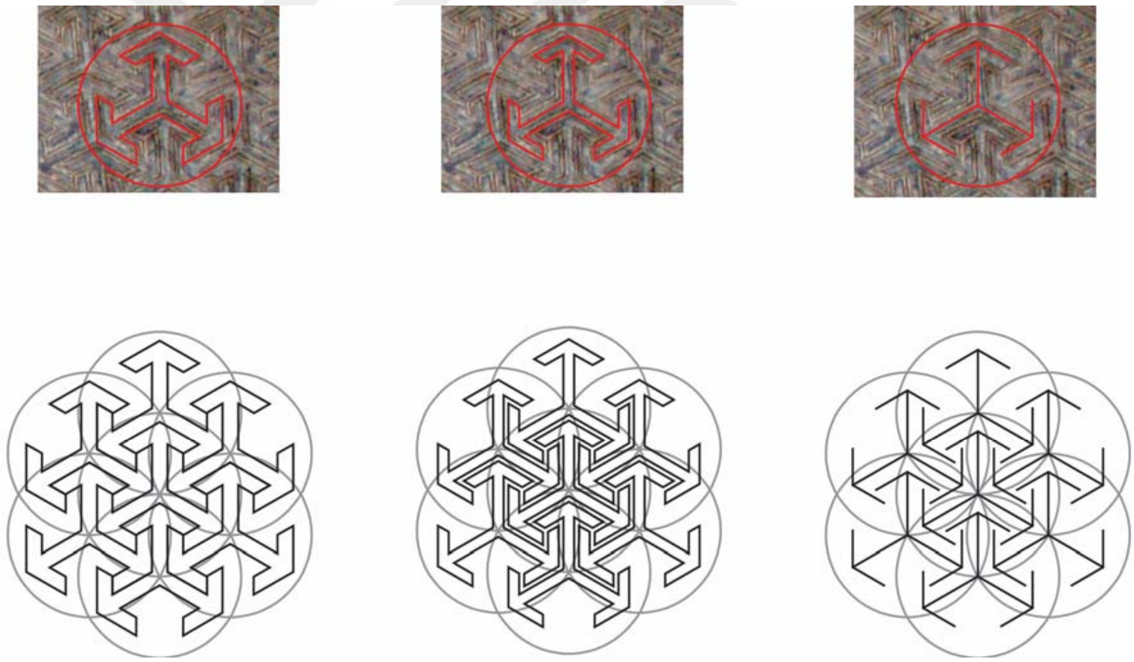

Figure 3.6 : From left to right: Drawings of the lines that go through the central axis

This result shows that different patterns can share common phases and initial shapes in their generation processes. Thus, in this study, I argue that various material shapes can be generated from a single initial shape based on the craftsmen's various actions in the making process, and this generative process can be formalized and visually computed with making rules. Figure 3.7 shows the transformations of the shapes by the making process for the two example patterns. In order to formalize the shape transformations by the making process, the boundaries of the resulting three-dimensional stone patterns on different levels were drawn as lines. The low areas of the pattern are represented as black colored surfaces, whereas the high areas are represented as white-colored surfaces.

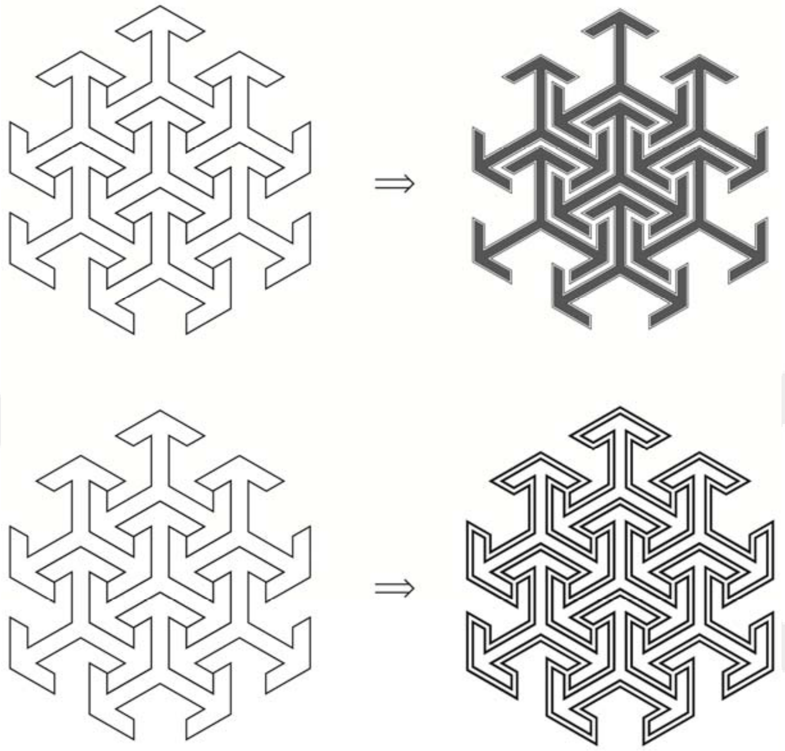

Figure 3.7 : Formalizations of shape transformations by the making process for Pattern I and II.

Moreover, there is an unusual case of the second pattern example. The pattern consists of seven stone blocks that were brought together, but not all the blocks have the same geometrical composition. The pattern on the second and the third blocks appears to be a little different than the other ones. The image and drawing of the different pattern in Figure 3.8 show that the overall geometrical composition look similar, but the six lines around the center were not engraved and instead another three lines were engraved additionally. The reason for this difference might be that these two blocks were damaged and remade by other craftsmen or the two blocks were just made by different craftsmen at the same time with the other blocks. In any case, this difference shows that the geometrical shapes of the discrete parts that are seen on the stone carved patterns are not predetermined, but were generated and emerged in the making process.

Figure 3.8 : Clockwise from top left: The image, the location and the drawing of the two blocks with the different geometrical composition.

The analysis of the generation of the initial shape, i.e. the guidelines, has been done by formalizing two alternative processes. Figure 3.9 shows the first alternative. In order to assist the reader's understanding of the process, two colors were used for formalizing the generative process. The black lines indicate the newly generated shapes and the gray lines indicate the previously generated shapes that are used as a reference. The shape computations represent the seven main transformation steps in the algebra of U_{12}. The first transformation is the addition of a hexagon on the circle grid by using the intersection points of circles as a reference. The second transformation is the multiplication of the hexagon by using the circle grid as a reference again. This multiplication is a kind of periodic tiling and can also be called tessellation. Since both the circular grid and the hexagon have a six-fold rotational symmetry, the transformation rule can be described both as radial or planar. The third transformation is the addition of a smaller circular grid with a six-fold rotational symmetry at the center by using the midpoints of the lines as a reference. The lines are the edges of the hexagon shapes. The fourth transformation is the addition of a hexagon around the circle at the center. The fifth transformation is the translation of the lines by the same particular distance in both directions. The distance, in this case, is equal to one-third of any edge of the lastly generated hexagon at the center. Thus, the distance can be calculated visually by using shapes. The sixth transformation is the definition of a particular shape. The transformation in this step can be described as the subtraction of the other lines that are erased at this point. Finally, the last transformation is the tessellation of the previously generated shape. In this case, the multiplied shape does not have six-fold rotational symmetry. Therefore, the transformation rule of this tessellation is clearly planar instead of radial.

The second alternative, shown in Figure 3.10, is represented as six main transformation steps that generate the same initial shape of the pattern example at the end. The first transformation is the addition of two circles. The radius of the circles is equal to the distance between the center of one circle and the perimeter of the other intersecting circle on the circular grid. The second transformation is the addition of two hexagons around the newly generated circles. The hexagon shape can be generated by dividing the circle into six equal parts and then drawing lines between the dividing points. The third transformation is the translation of the hexagon by the distance between the two hexagons. In this case, the translated hexagon shapes remain. The fourth transformation is the tessellation of the hexagon shapes on the circular grid. The fifth transformation is the subtraction of some lines by erasing them in order to generate the particular shape that is embedded in the existing shapes. Finally, the sixth transformation is the tessellation of the particular shape on the circular grid.

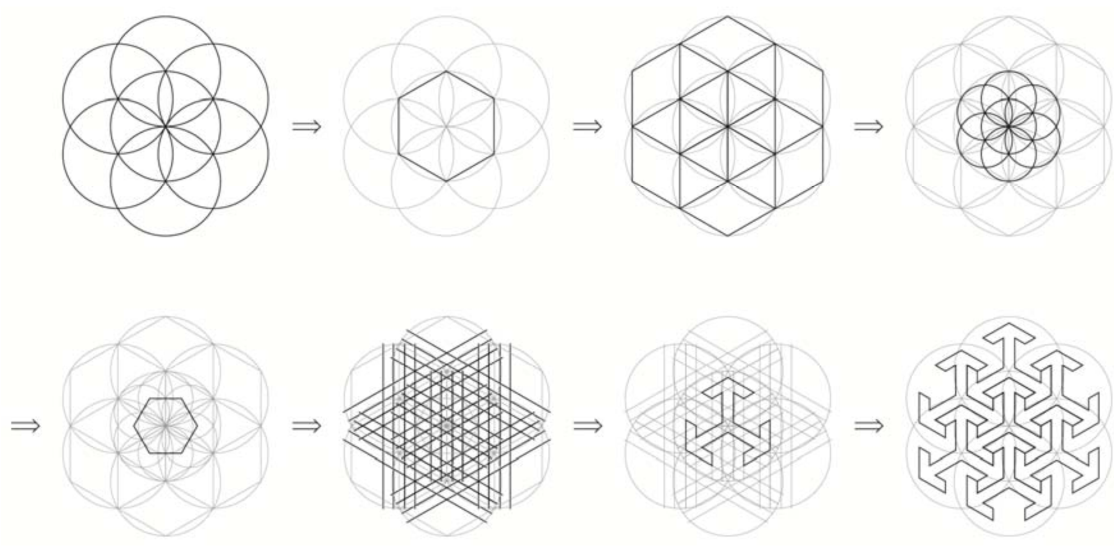

Figure 3.9 : Shape transformations of the first alternative process for generating the initial shape of Pattern I and II.

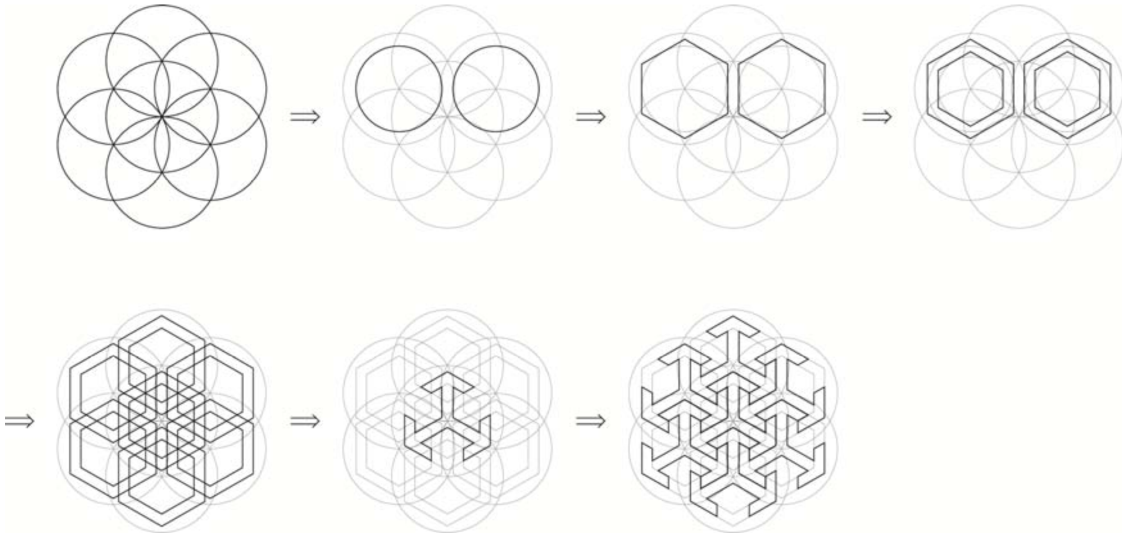

Figure 3.10 : Shape transformations of the second alternative process for generating the initial shape of Pattern I and II.

The third example is a group of patterns located at the corner of the same monument. Figure 3.11 shows an image of the patterns and the drawing of the whole composition. There are four types of patterns that were placed repeatedly, yet there is no formal rule regarding the order. Moreover, the patterns look like they were generated from the same original geometry and then differentiated by some rules in the process.

In order to analyze the formal relations between these four patterns, firstly the generation process of the first pattern from the top has been analyzed. Figure 3.12 shows the transformation steps for generating the pattern from a circular grid with eight-fold rotational symmetry. The first step is the translation of the circles by a particular distance towards their centers. The second step is the addition of hexagons inside the small circles with an eight-fold rotational symmetry. The third step is the addition of two squares inside the circle at the center. The fifth step is the subtraction, i.e. the erasion, of some parts to get the final geometry. The last step shows the transformation from the final geometry to the boundaries that can be seen on the stone carved pattern.

Furthermore, Figure 3.13 shows that the final geometry of the first pattern can be used as the initial shape for generating the other three patterns. The computations in the first row demonstrate the shape generation process of the second pattern. In this case, the first computation indicates the addition of sixteen lines that go through some of the intersection points on the existing shapes. The process continues with the subtraction of the interstitial lines between the intersection points to form the final geometry. The computations in the second row demonstrate the shape generation process of the third pattern. The process starts with the addition of a new circular grid with eight-fold rotational symmetry to the center of the pattern. The circles go through the corners of the square and the center point of the whole pattern. The computations in the third row demonstrate the shape generation process of the fourth pattern. This process also starts with the addition of a new circular grid with eight-fold rotational symmetry, but this time, the circles go through the intersection points of the squares and the center point of the pattern.

Figure 3.11 : The image and the drawing of Pattern III.

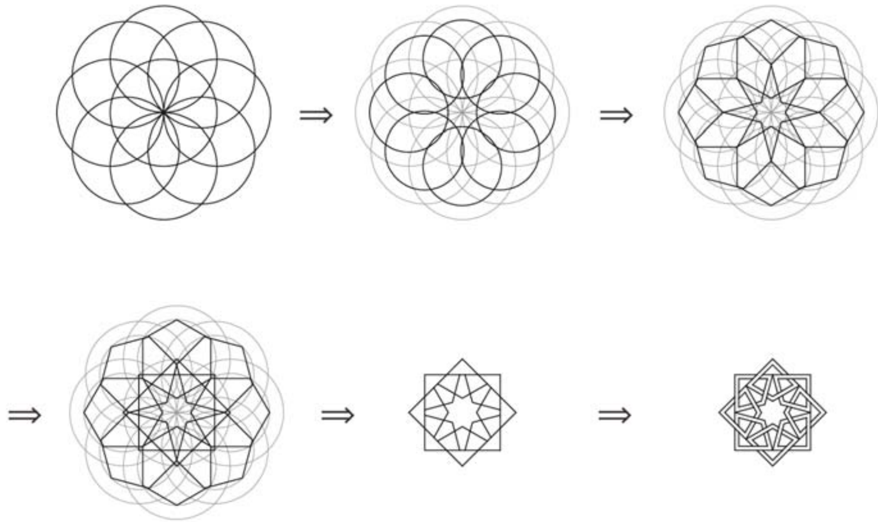

Figure 3.12 : The transformation steps for generating the pattern from a circular grid with eight-fold rotational symmetry.

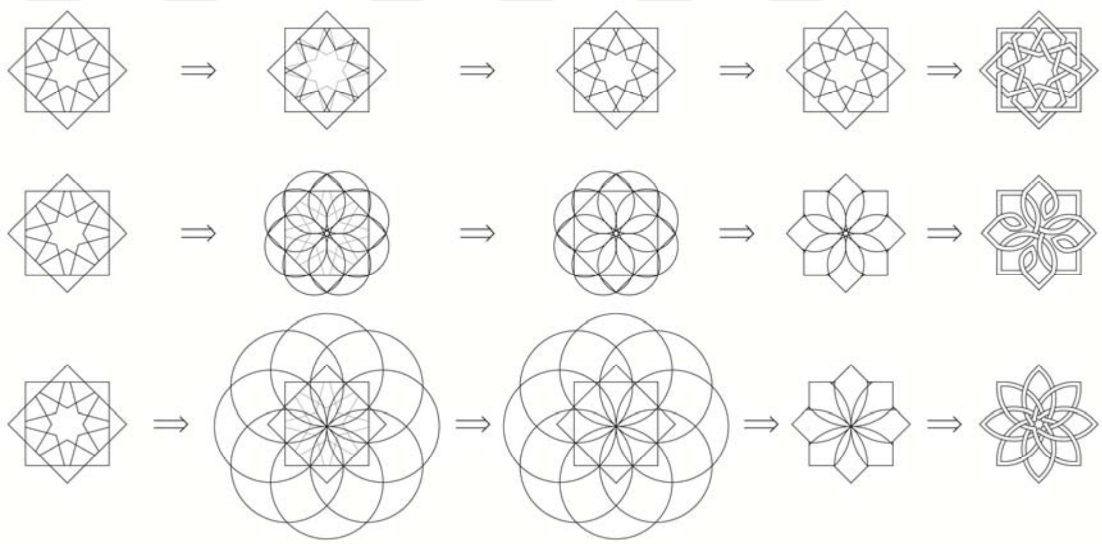

Figure 3.13 : Three pattern variations generated from the same initial shape.

The variations in Pattern III as a whole represents that, the variations based on emergent shape transformations from an initial shape were intended as part of the design generation process of Seljuk geometric patterns. Moreover, the presented method in Figure 3.13 is only one of the possible scenarios for generating this variation. For instance, the circular grids with eight-fold rotational symmetries in Figure 3.12 and Figure 3.13 (below) are similar. Therefore, the first pattern and fourth pattern can be derived from the same initial shape as well. All in all, it is hard to give a definite answer to the question of how these complex geometric patterns are generated, but the analysis shows that the transformations are relevant in their design generation.

Using shape computation formalism to analyze the generation process of the first three examples has led to the following conclusions. Firstly, there are emergent shapes derived in the process. Stiny's (2006) shape theory suggests that shapes are ambiguous in their nature. Shapes merge in a way that no parts can be distinguished without a particular definition and therefore infinite parts can be seen on a single shape. Thus, shape computations enable the emergence of new shapes and the development of new computations with them (Knight, 2003). An example of this kind of an emergence can be seen in the analysis of the second pattern example. Figure 3.14 shows that the particular transformation where the edges of different hexagon shapes are merged to generate a new and emergent shape. If there were no emergent shape generation in Seljuk geometric patterns, the design space of these patterns would be limited to the basic polygonal shapes. However, as Özkar (2014) puts it clearly, the limitless design space of Seljuk geometric patterns consist of various geometric shapes that emerge from how the artisan sees and transforms the shapes. Özkar also emphasizes that the reason for the emergence lies in the fact that, the patterns are non-figurative because of religious restraints in Seljuk-era and so, using abstract geometric shapes enables adaptability and flexibility to the emerging variations. In this study, the possible generation scenarios of the emergent shapes have been examined on selected pattern examples as a series of shape rules.

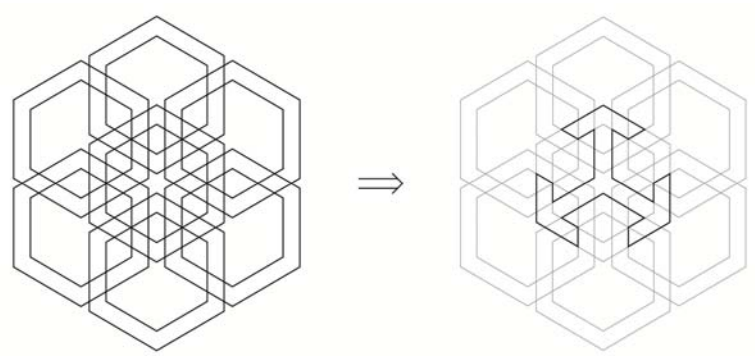

Figure 3.14 : The emergent shape that was generated from the merged hexagon shapes.

Secondly, general transformations can be identified as shape rules and these shape rules can form a shape grammar model for generating Seljuk geometric patterns. For example, Figure 3.15 shows three shape rules in the algebra U_{12} that were identified from the generation processes of the first two pattern examples. The first rule, shown in Figure 3.15a, is the addition of a hexagon inside a circle. The second rule, shown in Figure 3.15b, is the translation of a hexagon by a particular distance. The third rule, shown in Figure 3.15c, is the tessellation of a hexagon with a six-fold rotational symmetry.

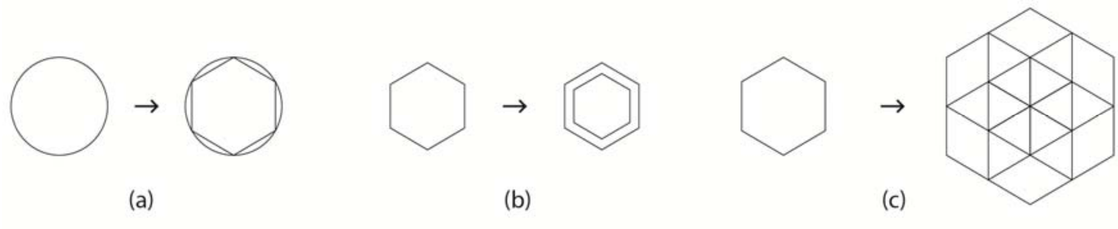

Figure 3.15 : Shape rules that indicate the addition (a), the translation (b) and the tessellation (c) of a hexagon.

Thirdly, emergent shapes can be generated by applying a simple rule to all parts of the pattern. For example, Figure 3.12c shows that multiplying and tessellating a hexagon with a six-fold rotational symmetry generates triangles and rhombuses. Özkar (2014) explains this phenomenon in Seljuk geometric patterns as a result of the various spatial relations that are based on the properties of the underlying circular grid such as repetition and symmetry.

Moreover, the group of patterns in the third example shows that circular grids can be used as part of the shape designs as well. In this case, fusing circles with polygons generate emergent shapes and variations. This example is also important for supporting the view that Seljuk geometric patterns are generated from circular grids.

Lastly, the shape rules can be more generalized in order to comprise e.g. the addition of all polygons, the translation of any shape by any distance or the tessellation of any shape with various rotational symmetries. In order to identify the generic and specific transformation rules for generating Seljuk geometric patterns, various patterns should be analyzed. In this study, fifty stone carved patterns from the Seljuk-era have been analyzed. The aim of this analysis is to show that each pattern can be generated by shape transformations in many different ways, and yet particular transformation methods can be explored and specified in order to reason about the generation process of the patterns. The patterns were drawn and analyzed with reference to the digital images. The images were obtained from the database of the scientific research project, which this thesis study has been involved in. The rest of this chapter highlights the seven transformation rules that were encountered during the study and the related examples.

3.1 Tessellation

Seljuk geometric patterns consist of repeating shapes. In this study, the tessellations of the repeating shapes are examined as shape rules. Applications of different tessellation rules to different initial shapes may result in various patterns.

The multiplied shapes may form various patterns based on the symmetry group of the circular grid. The patterns examined in this study show that, various rotational symmetries can be found on Seljuk geometric patterns. Examples show the tessellation rules on circular grids with six-fold (Figure 3.16), eight-fold (Figure 3.17), twelve-fold (Figure 3.18) and ten-fold (Figure 3.19) rotational symmetries. Another example shows the unfolded and folded states of a three-dimensional tessellation (Figure 3.20). Moreover, some examples obtain multiple rotational symmetries. For instance, Pattern VIII in Figure 3.21 appears to have eight-, sixteen- and twentyfour-fold rotational symmetries from the center outwards. These three different tessellation rules were used on two different initial shapes, which are hexagon and octagon. This unique example shows that various geometric patterns can be generated by using different tessellation rules. Therefore, the use of different tessellation rules appears to be one of the main generative processes for producing the Seljuk geometric patterns.

Additionally, the tessellated shapes do not need to be in contact with each other since they are constructed based on the underlying circles. For example, Pattern IV in Figure 3.16, features tessellated hexagons on a circular grid with six-fold rotational symmetry and the hexagons do not share any intersection points. Some other examples show that parts of the regular geometric shapes might be translated towards the center of the circles or subtracted. In these cases, the repeated shapes on the finished geometric compositions may not be in contact as well. Yet, the overall compositions preserve the radial symmetry, since their center points are always identical with the circles they are inscribed in.

Moreover, a general assumption can be made about the relation between the rotational symmetry of the polygons inscribed in each circle and the overall rotational symmetry of the circular grid. The patterns examined in this study show that the use of same rotational symmetry for each repeated geometric shape and the overall grid is a common method. For instance, the hexagons are tessellated with six-fold rotational symmetry in Pattern IV (Figure 3.16); the octagons are tessellated with eight-fold rotational symmetry in Pattern V (Figure 3.17); the hexagons are tessellated with twelve-fold rotational symmetry in Pattern VI (Figure 3.18) and the pentagons are tessellated with ten-fold rotational symmetry in Pattern VII (Figure 3.19). There are also examples of unlikely situations. For example in Pattern VII, there are also hexagons at the center of the pattern and as a result of this variety, a shape that looks like a half bow-tie has been generated between the hexagons and pentagons.

After all, tessellations in Seljuk geometric patterns appear to be a specific kind of tiling where neither the tiles nor the whole are predetermined but can be further developed. Applications of different tessellation rules based on different circular grids, rotational symmetries, and initial shapes may result in emergent wholes as geometric designs that are constructed from basic geometric parts.

3.2 Addition and Subtraction

In this study, some of the shape transformations in design generation of Seljuk geometric patterns have been specified as addition and subtraction rules. The examples include additions and subtractions of single lines or polygons.

For example, an addition rule can be formalized to describe one of the transformations in the shape generation process of Pattern X (Figure 3.22). The pattern is a rosette design at the entrance of Kayseri Gevher Nasibe Hospital (Çifte Kümbet), which was built at the beginning of the 13th century. The design can be generated by adding three intersecting lines inside a hexagon that divide the hexagon into six equal triangles. Afterward, these three lines can be translated towards both sides to finish the design. Another addition rule can be formalized to describe a transformation in the shape generation process of Pattern XI (Figure 3.23). The pattern is located on the pediment at the entrance of the Hospital of Amasya, which was built at the beginning of the 14th century. In this case, a square is added at the center of four dodecagons in four-fold rotational symmetry. This example shows that complex geometric compositions can be constructed by adding up various geometric shapes. Some other example patterns can be generated by subtraction. For instance, a subtraction rule can be formalized to describe one of the transformations in the shape generation process of Pattern XII (Figure 3.24). The pattern is located on an arch at

Mahperi Huand Hatun Complex in Kayseri, which was built in the first half of the 13th century. The subtraction rules show the removal of specific lines from two intersecting squares. In another example, the design layout of Pattern XIII can be generated with a subtraction rule (Figure 3.25). The pattern is another rosette design from the entrance of Kayseri Gevher Nasibe Hospital. The shape generation scenario starts with constructing a pattern with ten decagons in a ten-fold rotational symmetry. A large part of the pattern is then erased to generate the final design. The trimming of the large part of the geometric composition makes the initial shape unrecognizable and enables the generation of an emergent new pattern design.

3.3 Translation

Shapes are translated when they are transferred from one place to another. This transformation can be formalized in the form of shape rules by using labels and can be applied to any shape. In this study, the focus is on the translation of polygons inscribed in circles. When drawing on a stone surface using only basic tools such as compass and straightedge, polygons can be translated by first translating the reference points to draw the polygons through. For instance, if the reference points are translated towards the inside of a polygon, the new polygon that goes through these points will be smaller than the first one. In this case, the transformation can also be formalized as a scaling rule. Moreover, such transformations are not done in one step, instead, they consist of many steps. Yet, the rules in this study indicate the transformation from the initial shape to the final shape in one step in order to formalize and compare different types of transformations that may generate the patterns.

For example, Pattern XIV at the entrance of the Tomb of Mama Hatun in Erzincan consists of hexagons inscribed in circles (Figure 3.26). The hexagons do not fit directly in the initial circular grid, that is geometrically constructible. The circles that the hexagons can be inscribed in are smaller than the initial ones. Therefore, in this shape generation scenario, each circle is transferred towards its center with a particular distance. This transformation can also be called scaling, but the construction of the geometry consists of transferring moves and therefore it is considered as a kind of translation in this study. As a result of this transformation, six-pointed geometric stars are generated at the center of the circles. This shape generation process presents how simple transformation rules can generate emergent shapes on geometric patterns.

3.4 Extrusion

Linear extrusion appears to be one of the typical transformation methods for constructing geometric patterns. Extrusions can also be regarded as additions, but in this study, they are specified as a particular transformation.

One of its typical uses can be seen in the shape generation analysis of Pattern XV (Figure 3.27). The pattern is another rosette design from the entrance of Kayseri Gevher Nasibe Hospital. The geometrical composition can be generated from ten pentagons in a ten-fold rotational symmetry. Then, all edges of each pentagon can be extruded towards both sides. The rule that was applied at this step is formalized as an extrusion rule (Figure 3.27). As a result, a ten-pointed geometric star can be generated at the center of the pattern.

In another example, the layout geometry of Pattern XVI can be generated by using a particular extrusion rule (Figure 3.28). The pattern is located at the entrance of Mahperi Huand Hatun Complex in Kayseri. The layout can be derived from squares and octagons within a four-fold rotational symmetry. Then, squares and octagons can be connected through linear extrusion s. As a result of this transformation, rhomboids are generated on the pattern. This extrusion can be formalized as the extrusion of the octagon towards the squares and vice versa. The related extrusion rule that was used in this shape computation can be seen in Figure 3.28.

The last example for this transformation is Pattern XVII (Figure 3.29). The pattern is located on the façade of the Tomb in Kayseri area (Döner Kümbet), which was built in the second half of te 13th century. In this instance, the pattern is generated by connecting octagons through linear extrusions. The extrusion rule can be formalized in many ways. One way to describe this transformation is the extrusion of each edge of the octagon towards the surrounding circles always in the right direction and is represented in the Figure 3.29. In any case, the transformation results in the generation of a rich geometric composition, that appears to be composed of repeating unusual geometric figures. The related extrusion rules that were used in this shape computation can be seen in Figure 3.29.

3.5 Rotation

Some Seljuk geometric patterns can be generated by rotating particular geometric shapes. Rotated shapes fuse together to generate emergent and unusual geometric compositions. For example, a possible way to generate Pattern XVIII can resemble this type of transformation (Figure 3.30). The pattern is located at the entrance of the Tomb of Rabia Hatun in Erzurum. Firstly, the boundaries of the carved parts on the surface look like two pentagons that are attached together by sharing an edge. Yet, the angle between the lines is not equal to the interior angle of a pentagon. Secondly, the pattern has a four-fold rotational symmetry. Therefore, after several experiments were conducted to fit a regular polygon on a four-fold circular grid, a possible shape generation process has been formalized by using a rotation rule. In this instance, the pattern is derived from squares. Since the pattern layout has a four-fold rotational symmetry, the use of squares appears likely. In the next step, two intersecting lines are added inside the squares. Lastly, the intersecting lines are rotated to construct the final layout geometry of the pattern. This transformation can be applied by translating the lines on a drawing or drawing the repeating parts on separate papers and then rotating them. The related rotation rules that were used in this shape computation can be seen in Figure 3.30.

Another example pattern that can be generated by rotation is Pattern XIX (Figure 3.31). The pattern is located on a muqarnas unit at the entrance of the Tomb of Mama Hatun. The rotational symmetry is six-fold. The boundaries of the carved parts are six-pointed geometric stars, hexagons and, similar to the previous example, a shape that looks like two pentagons attached together by sharing an edge. Moreover, the hexagons are also stand in a six-fold rotational symmetry around the six-pointed stars at their centers. The corners of each hexagon within the six-fold rotational symmetry fits in the twelve pointed shapes around the six-pointed stars. The shape generation process produced in this study starts with drawing two intersecting hexagons inscribed in circles with the six-fold symmetry. Additionally, the circles around the hexagons are smaller than the initial circular grid and therefore are translated before drawing the hexagons. Finally, some parts are subtracted from the overall composition to finish the layout design of the pattern. The related rotation rule that was used in this shape computation can be seen in Figure 3.31.

The last example in this study to represent pattern generation by rotation is Pattern XV (Figure 3.32). The pattern is located on an engaged column at the entrance of Mahperi Huand Hatun Complex in Kayseri. Unlike many other Seljuk geometric patterns, the boundaries of the carved parts do not resemble any regular polygon directly. There are parallel lines that stand in a six-fold rotational symmetry. The eyes can pick Z-shaped figures in a six-fold rotational symmetry as well. However, these lines do not fit on a hexagon as they are. One way to generate this pattern might be to start with drawings hexagons inscribed in smaller circles inside the circles of a six-fold circular grid. Then, the hexagons are rotated and three intersecting lines are drawn inside each hexagon. Thus, all hexagons are divided into six equal triangles. In the final step, some parts are subtracted from the overall geometry to finish the layout design of the pattern. The related rotation rule that was used in this shape computation can be seen in Figure 3.32.

3.6 Curving

In this study, curving refers to the transformation, where curvilinear shapes are constructed by drawing a continuous line through the corners of a linear geometric shape. In this way, variable complex curvilinear patterns can be derived from polygonal geometric constructions. For example, the shape generation of Pattern XVI at the Tomb of Mama Hatun can be described as such a transformation (Figure 3.33). The process starts with generating a circular grid with eight-fold rotational symmetry. The squares are then inscribed in the circles. Some parts are subtracted from the overall composition. Lastly, curvilinear shapes are constructed by using the corner points on the previous geometric composition as reference points. The related curving rule that was used in this shape computation can be seen in Figure 3.33.

All in all, this chapter has analyzed possible generative methods for producing the design layouts of Seljuk geometric designs that were carved into stone. The possible shape transformations in the analyzed generative processes have been represented in the form of shape rules. The rules have been classified as seven different generic rule schemas which are tessellation, addition, subtraction, translation, extrusion, rotation and curving. This rule-based method could be further developed as part of a comprehensive shape grammar for the Seljuk geometric patterns. However, this thesis does not engage with developing a shape grammar for Seljuk geometric patterns, which would require extensive examinations of more example patterns. On the other hand, various possible transformations examined in this study suggests that design generation of the patterns can be formalized in discrete phases. This method helps to reveal formal relations between various patterns, as well as to integrate infinite possible other geometric compositions in the design generation of new patterns.

The main purpose of this chapter was to analyze the relationship between shapes and their making process. The results provided the layout geometries that can be used as the initial shape in the making process., which will be presented in the next chapter.

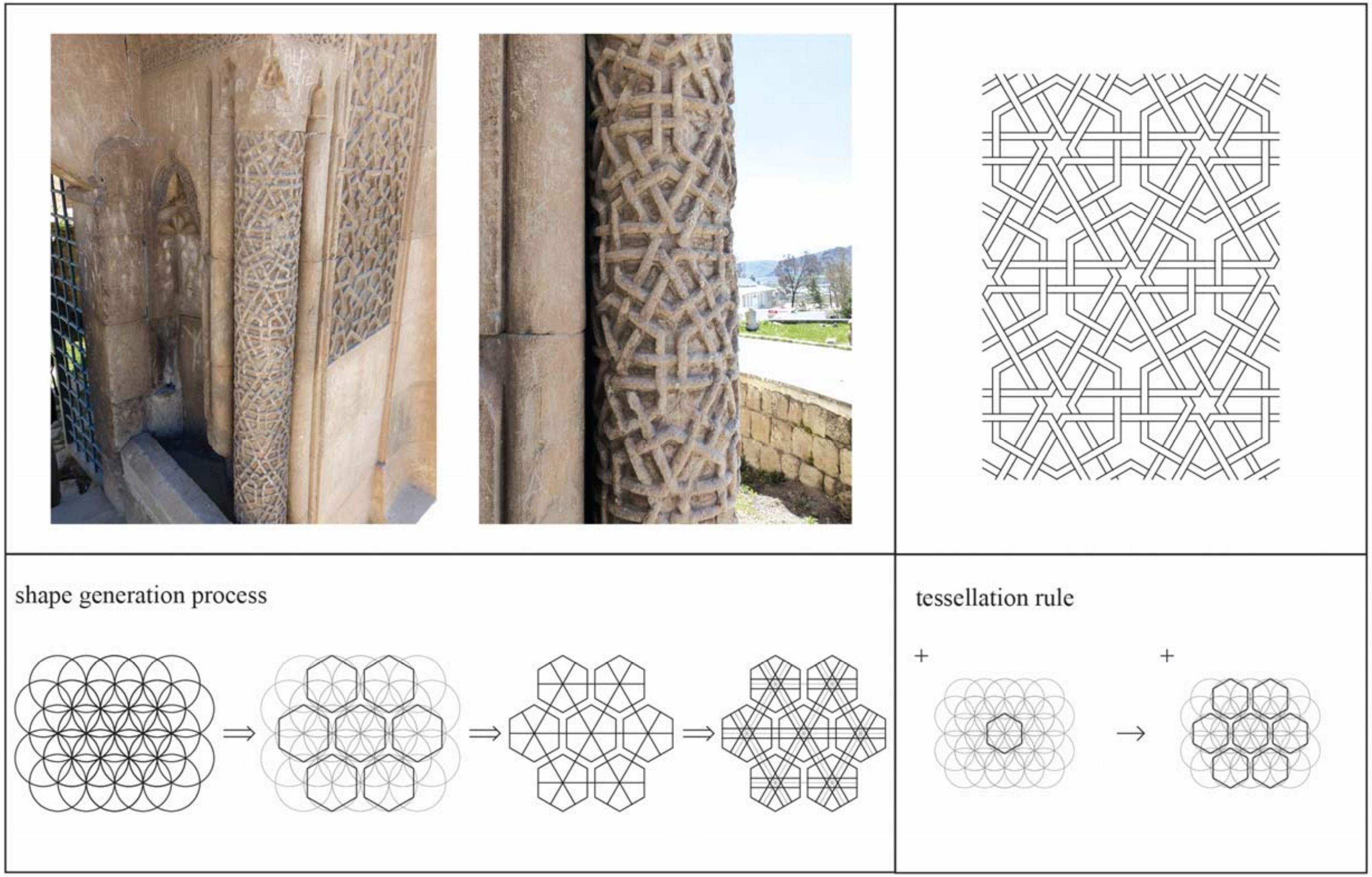

Figure 3.16 : Shape generation process and the tessellation rule of Pattern IV.

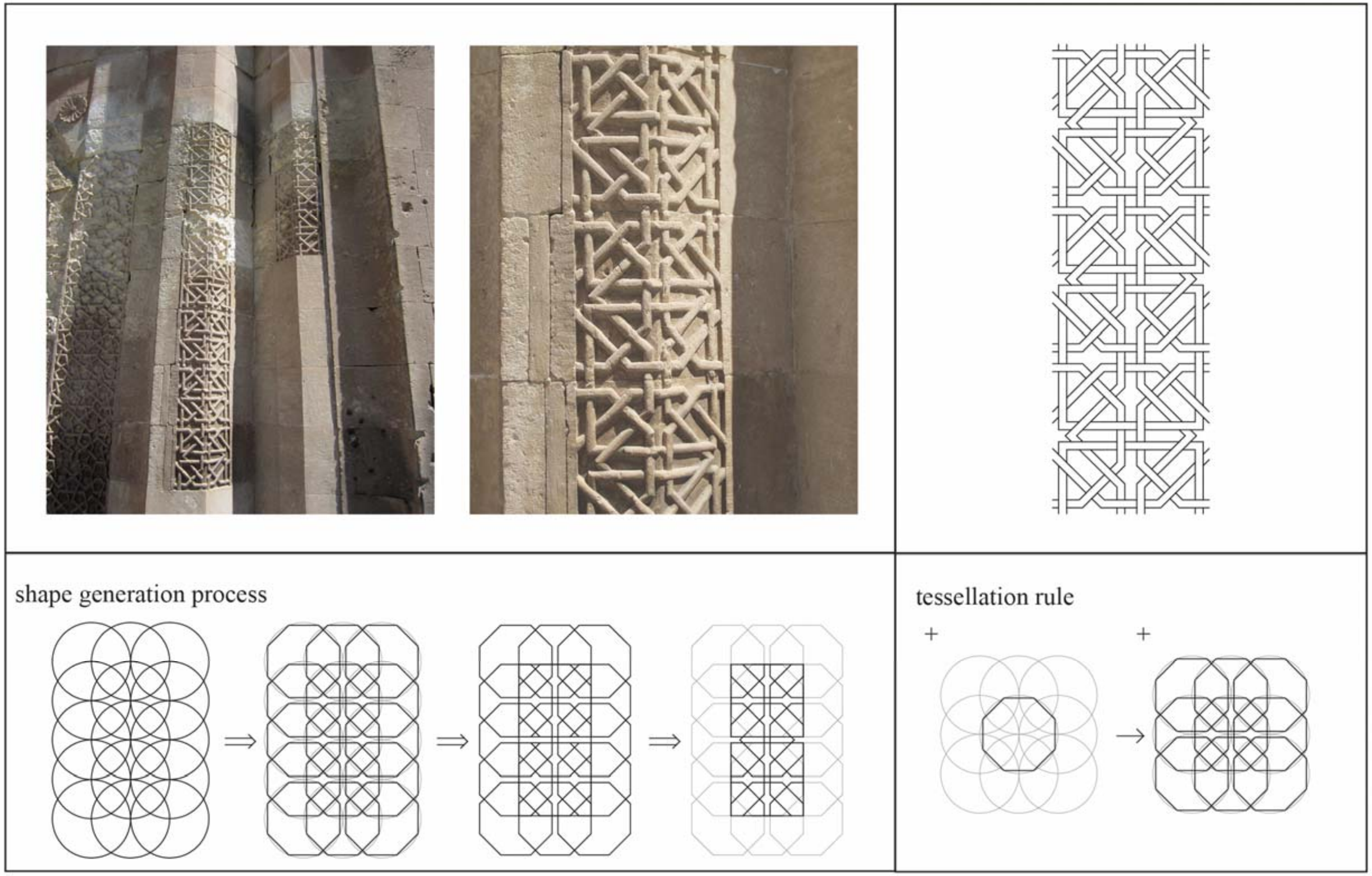

Figure 3.17 : Shape generation process and the tessellation rule of Pattern V.

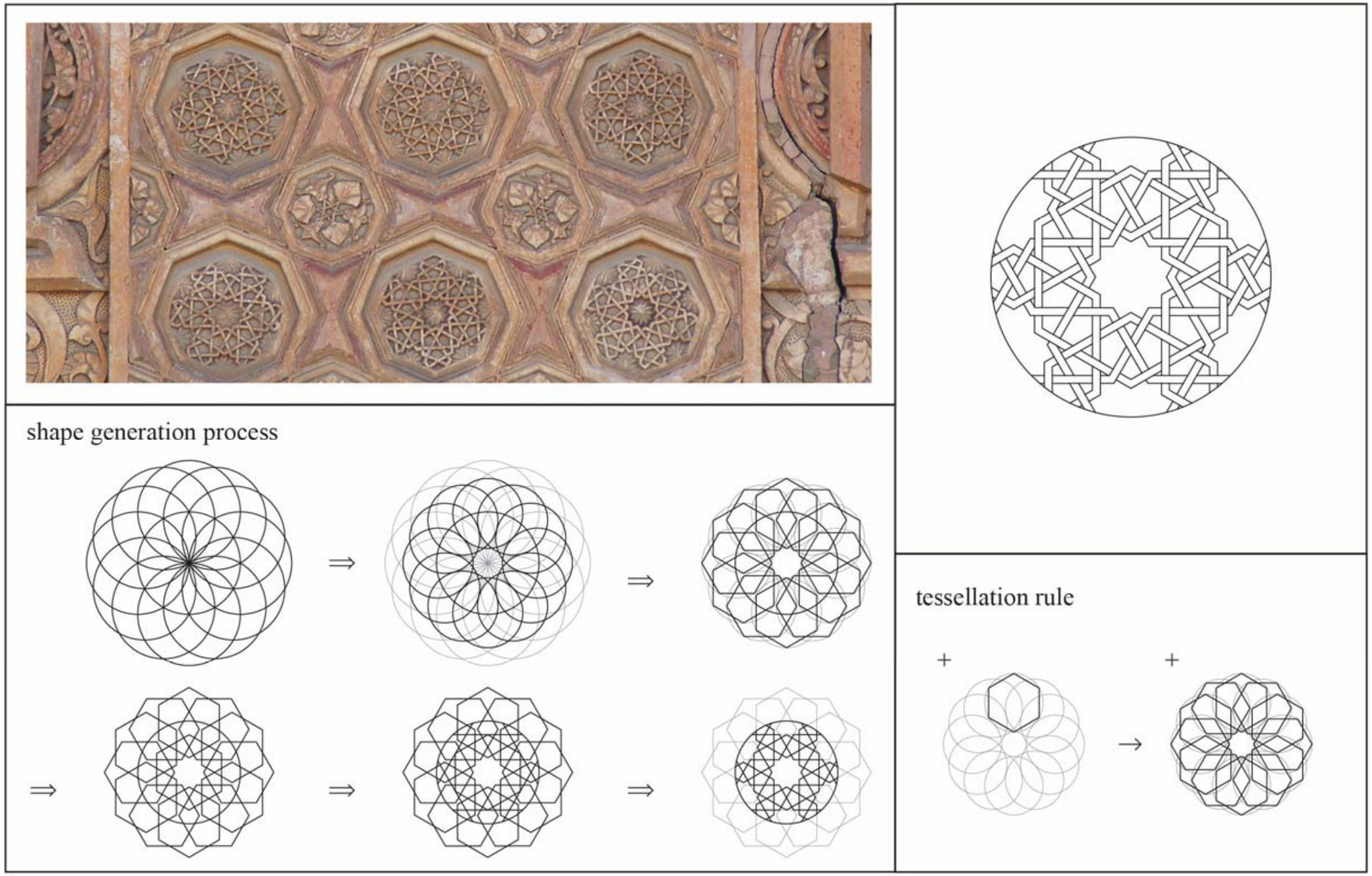

Figure 3.18 : Shape generation process and the tessellation rule of Pattern VI.

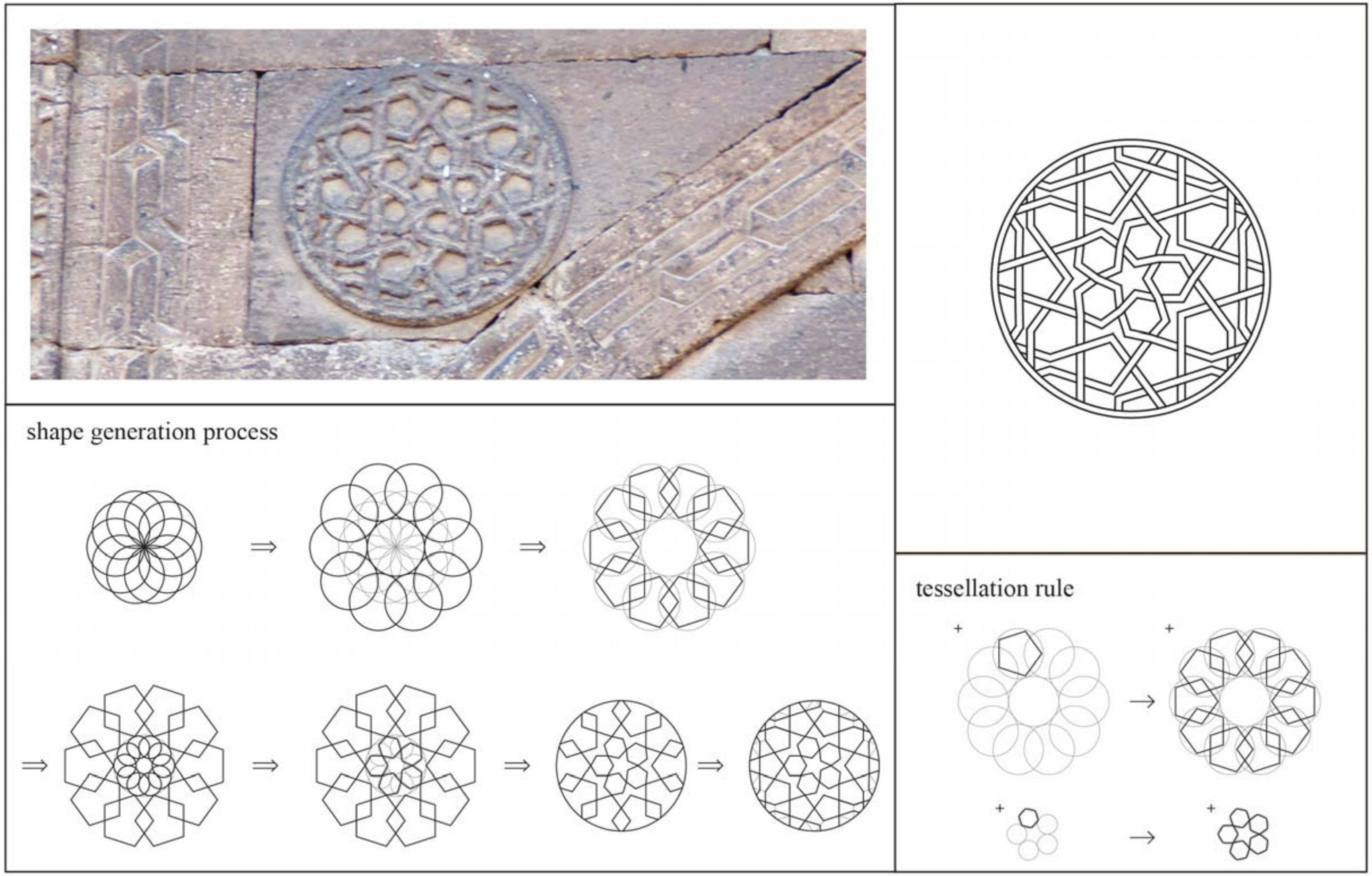

Figure 3.19 : Shape generation process and the tessellation rules of Pattern VII.

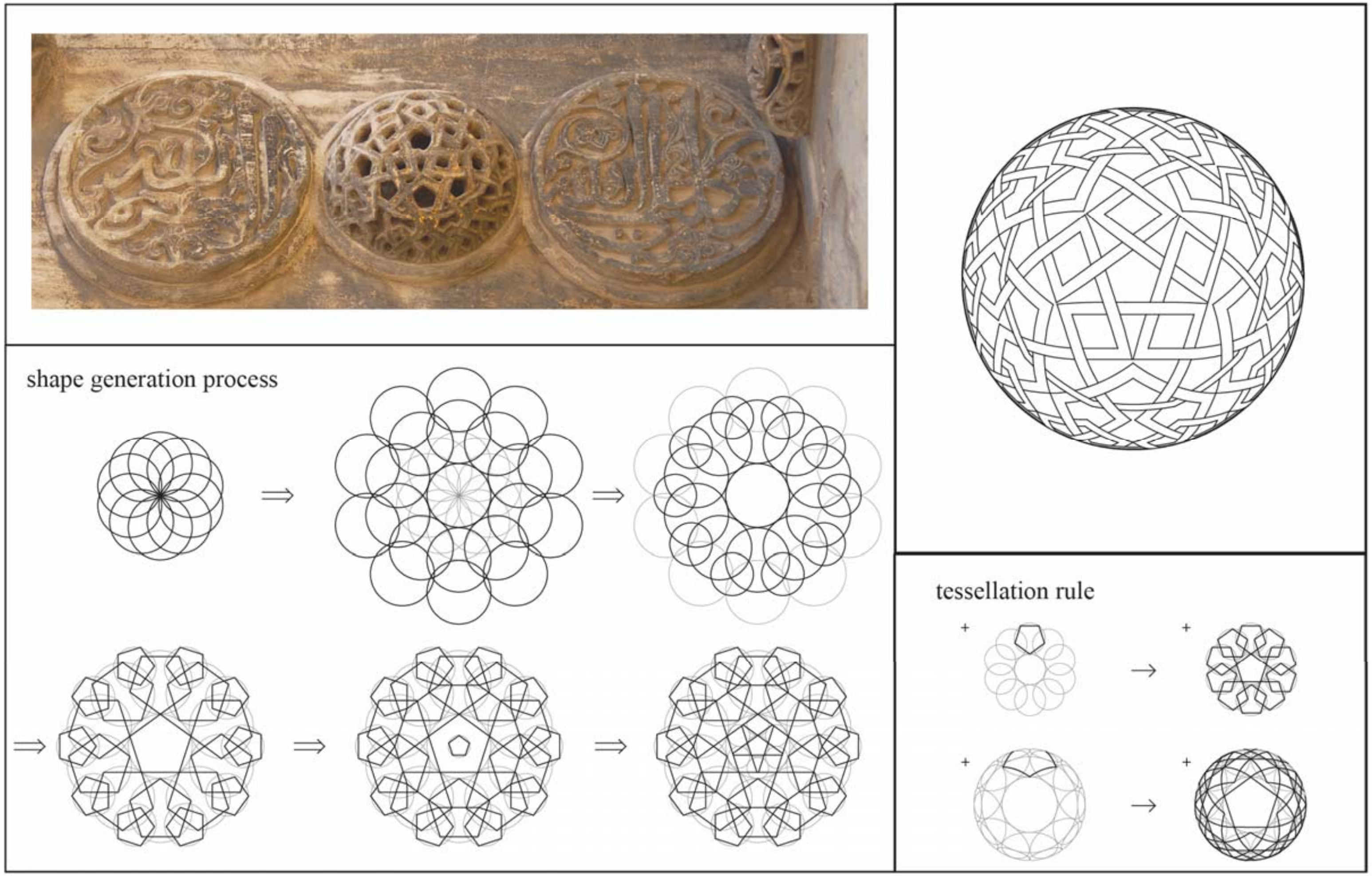

Figure 3.20 : Shape generation process and the tessellation rules of Pattern VIII.

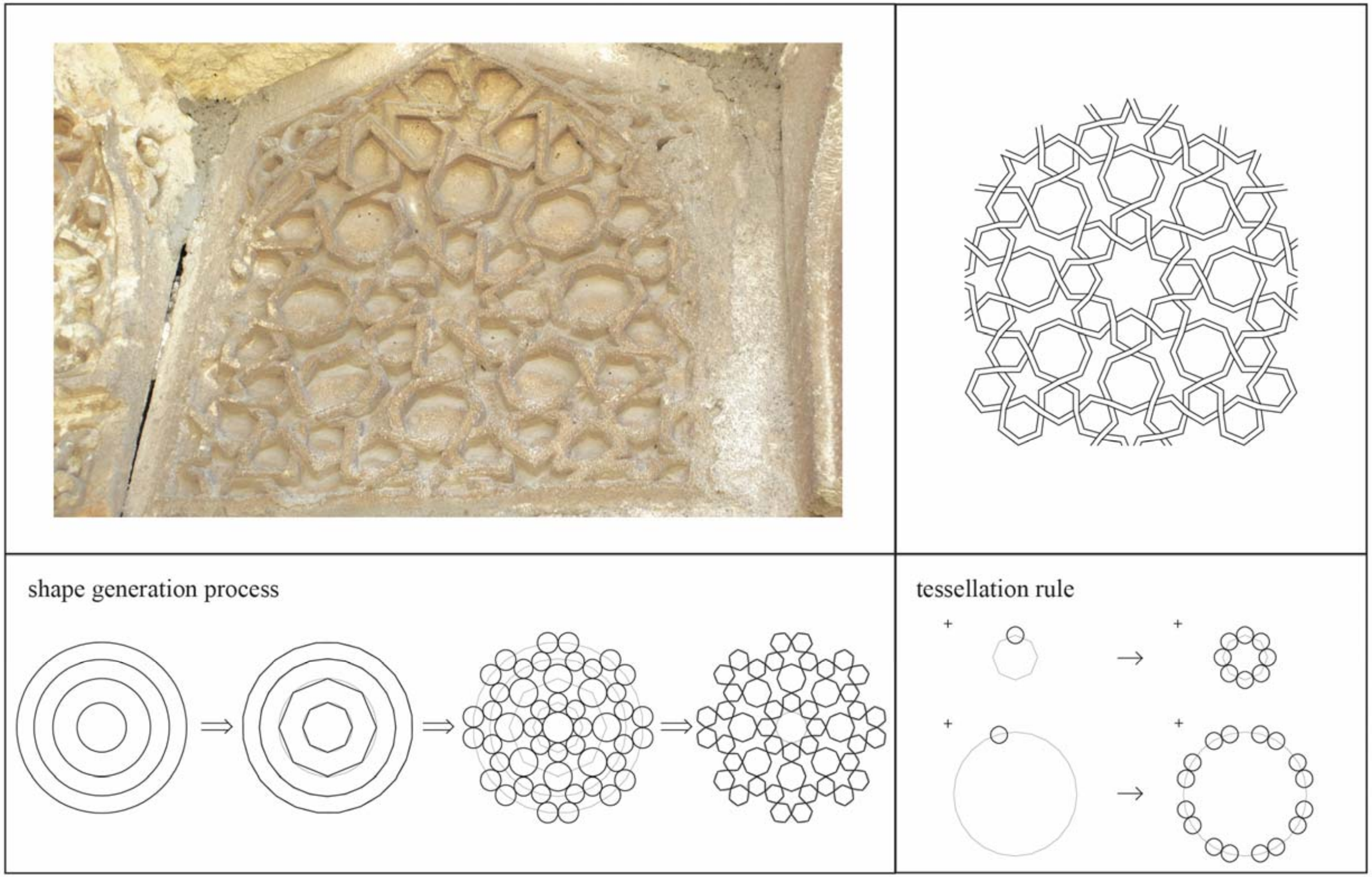

Figure 3.21 : Shape generation process and the tessellation rules of Pattern IX.

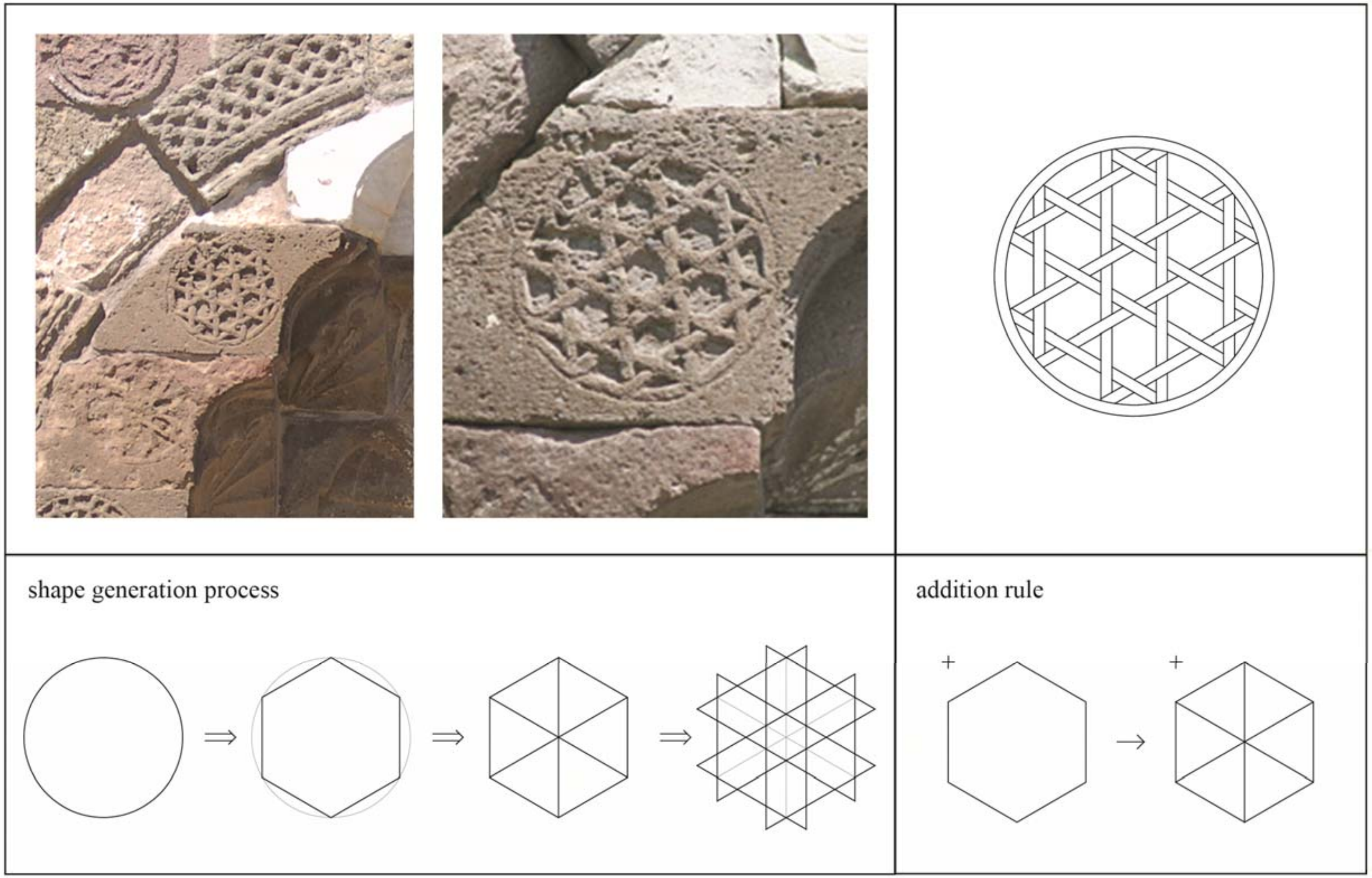

Figure 3.22 : Shape generation process and the addition rule of Pattern X.

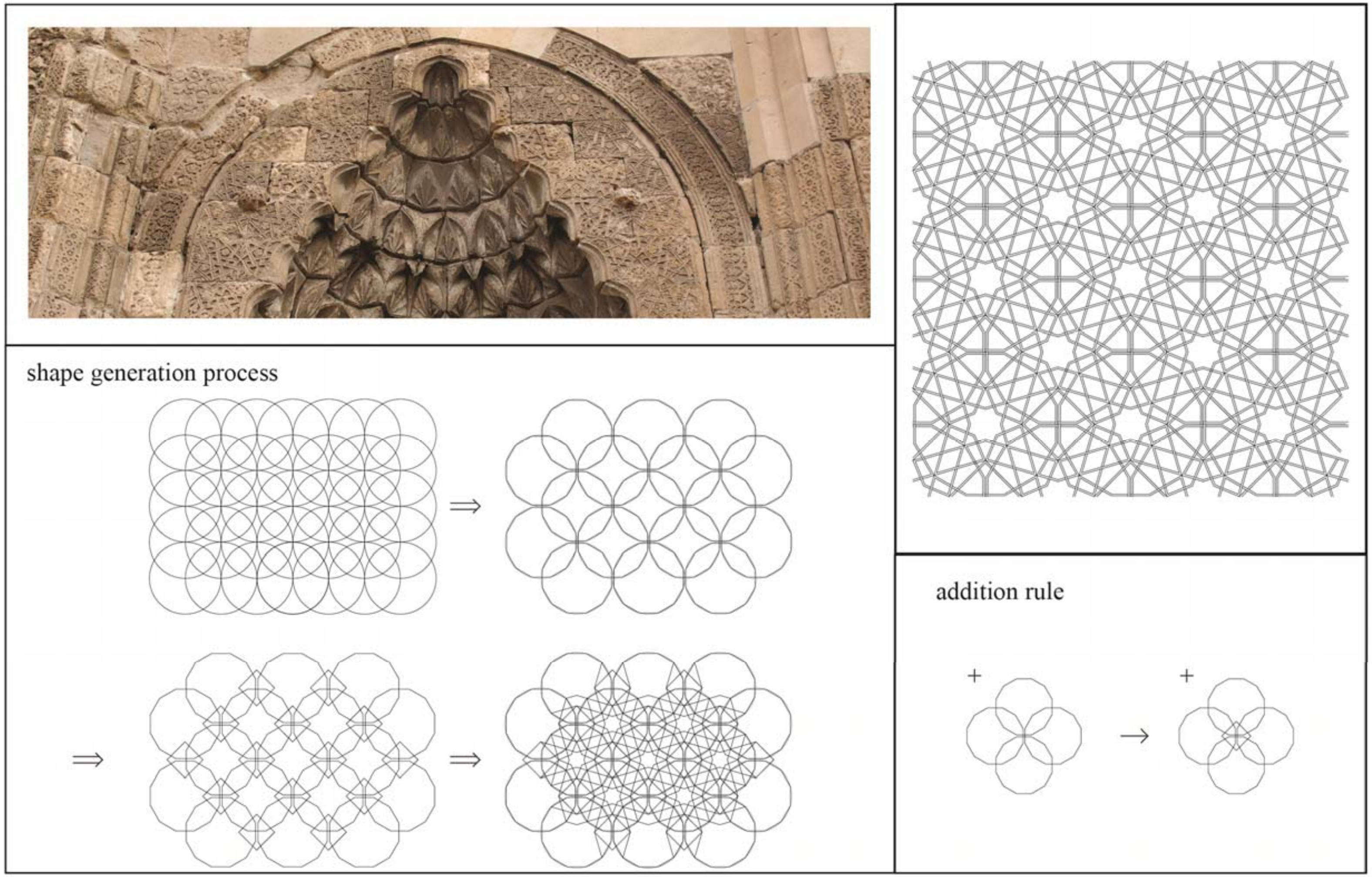

Figure 3.23 : Shape generation process and the addition rule of Pattern XI.

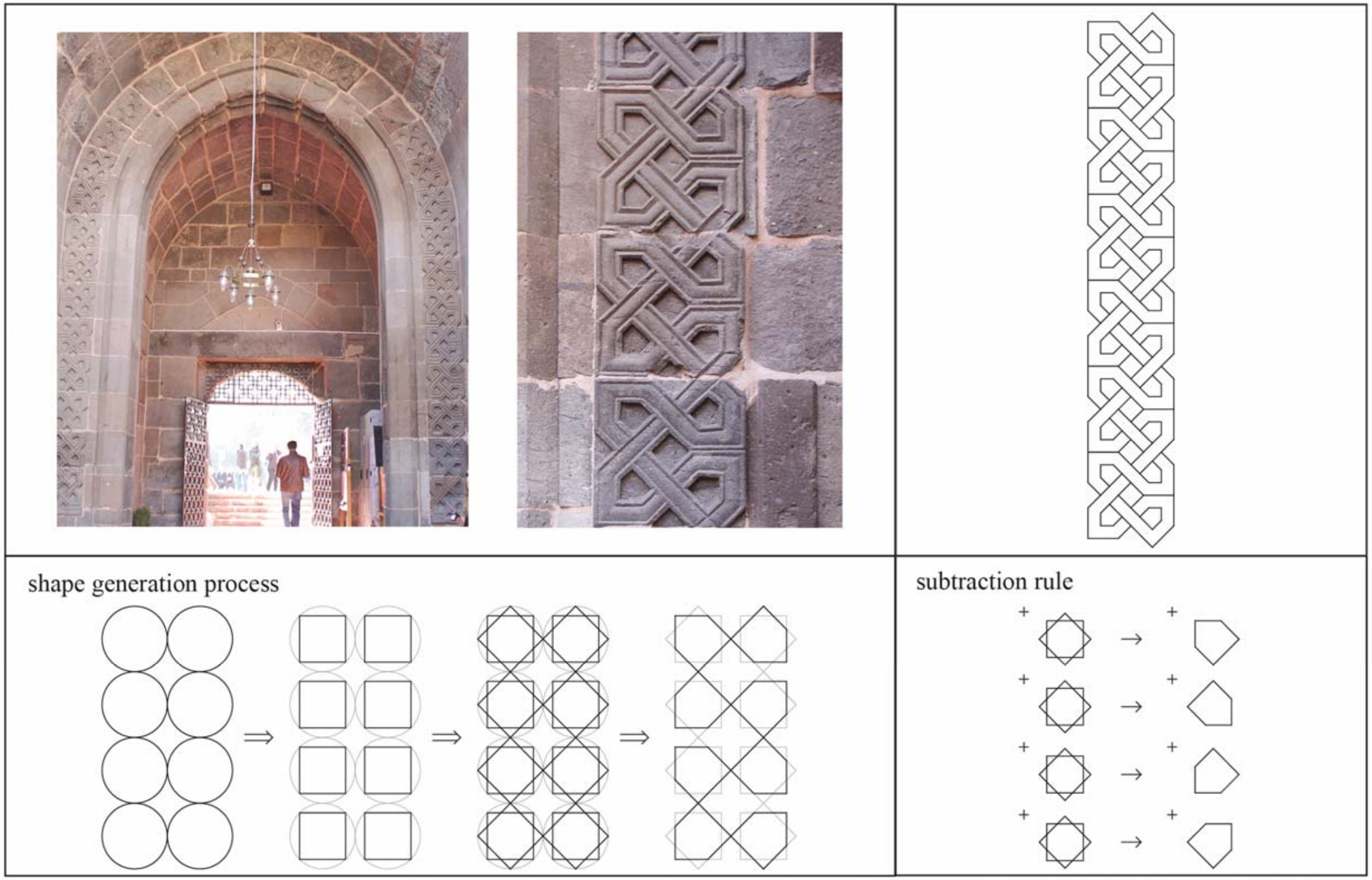

Figure 3.24 : Shape generation process and the subtraction rule of Pattern XII.

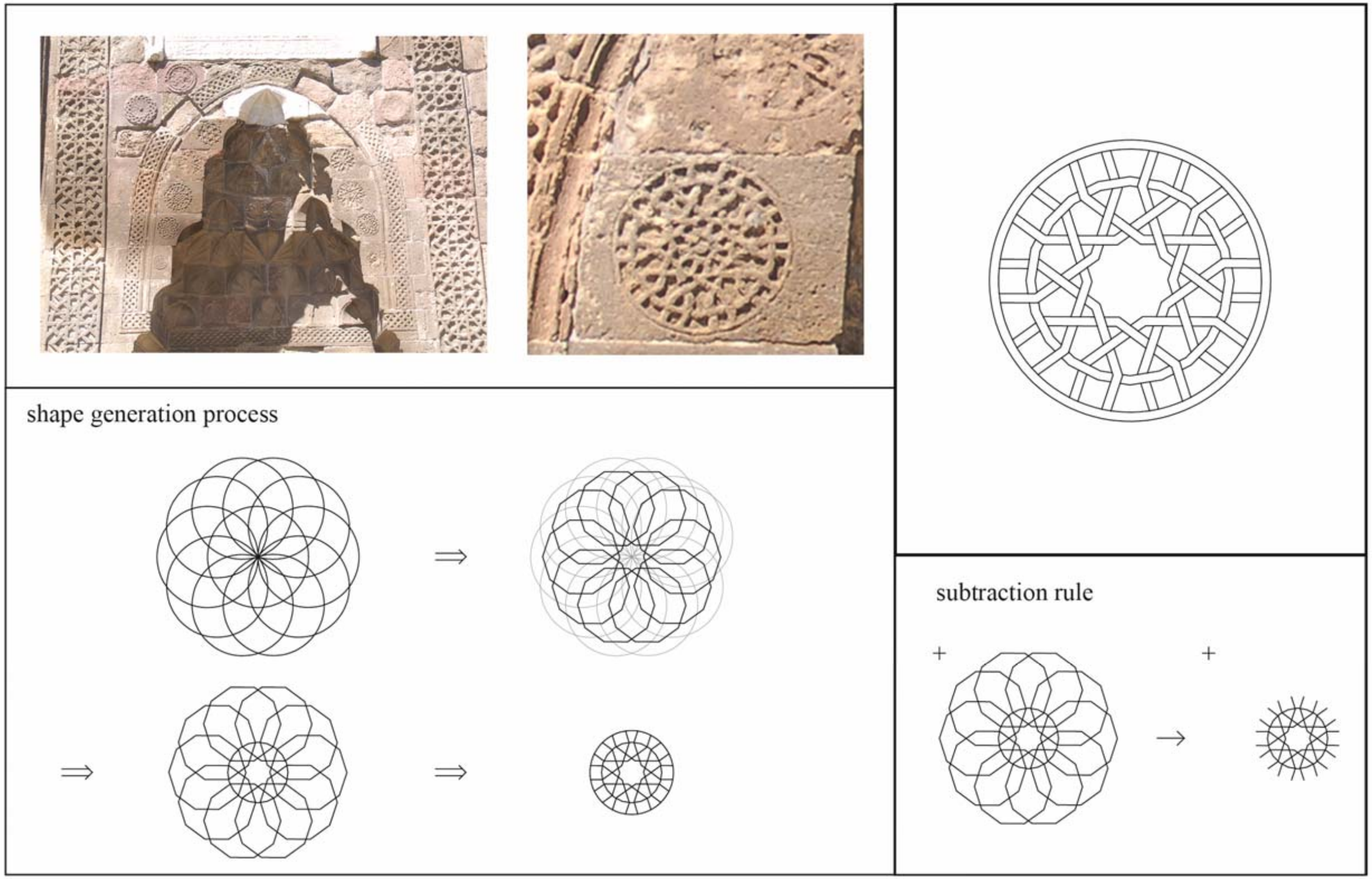

Figure 3.25 : Shape generation process and the subtraction rule of Pattern XIII.

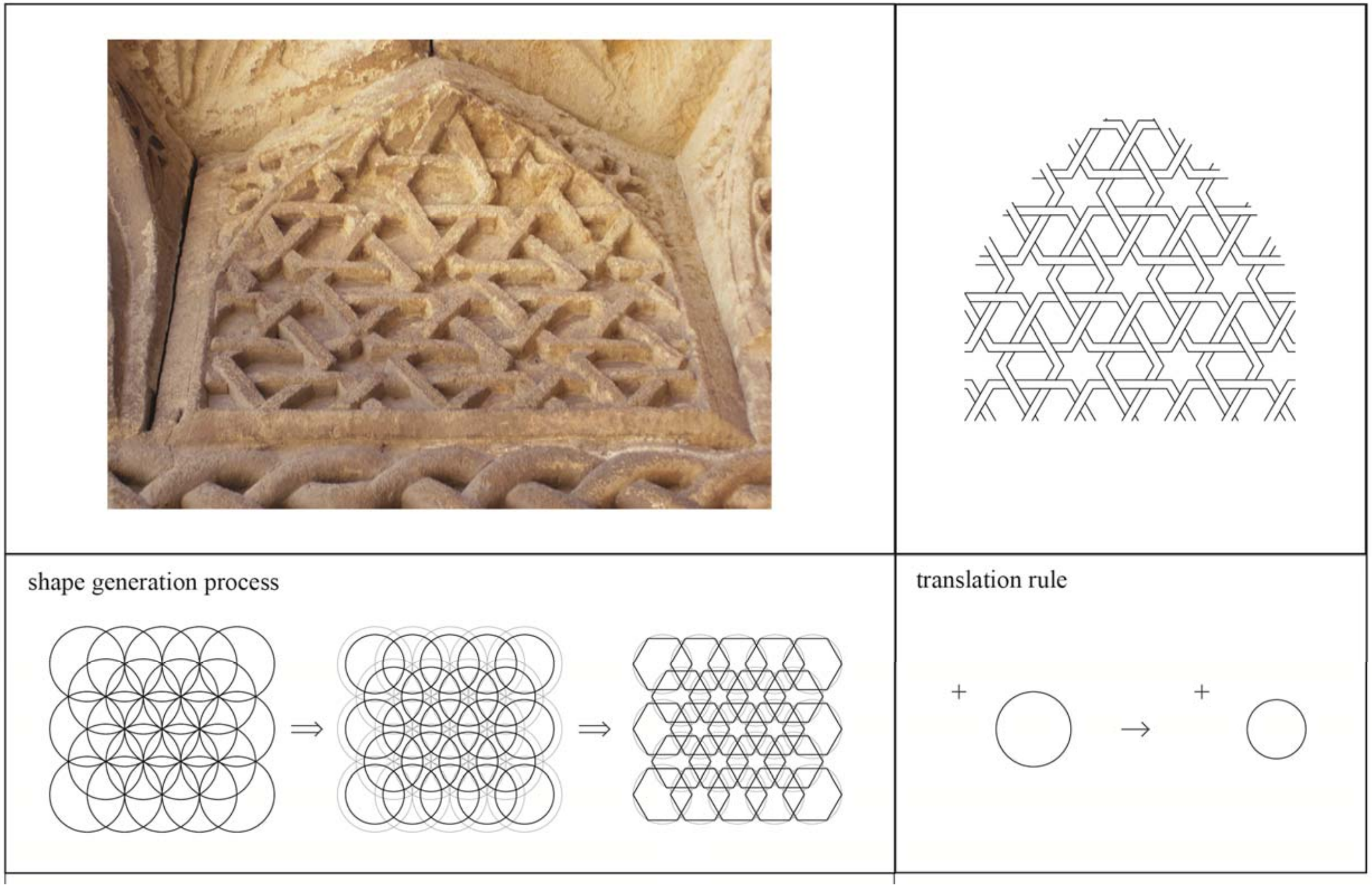

Figure 3.26 : Shape generation process and the translation rule of Pattern XIV.

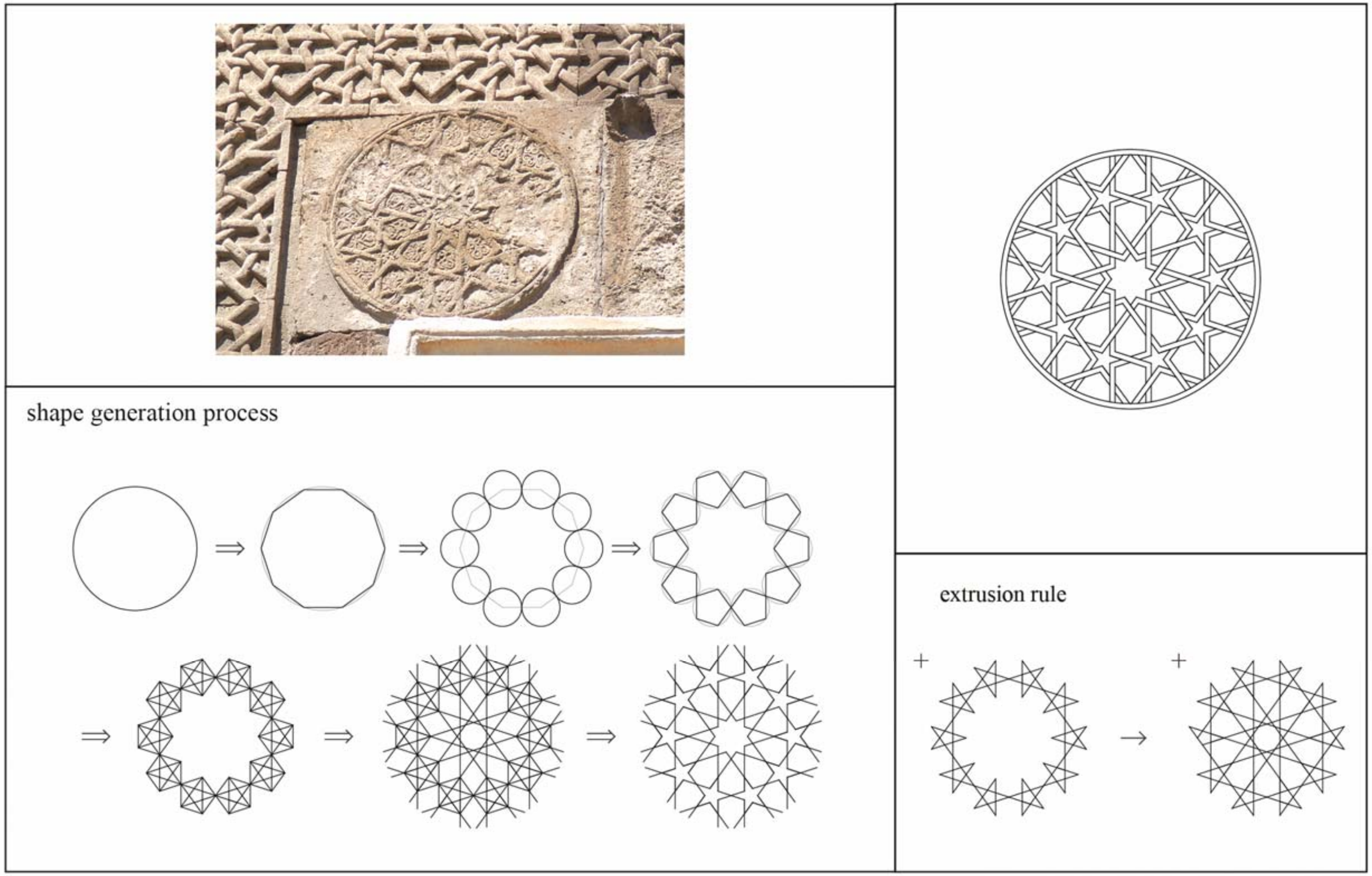

Figure 3.27 : Shape generation process and the extrusion rule of Pattern XV.

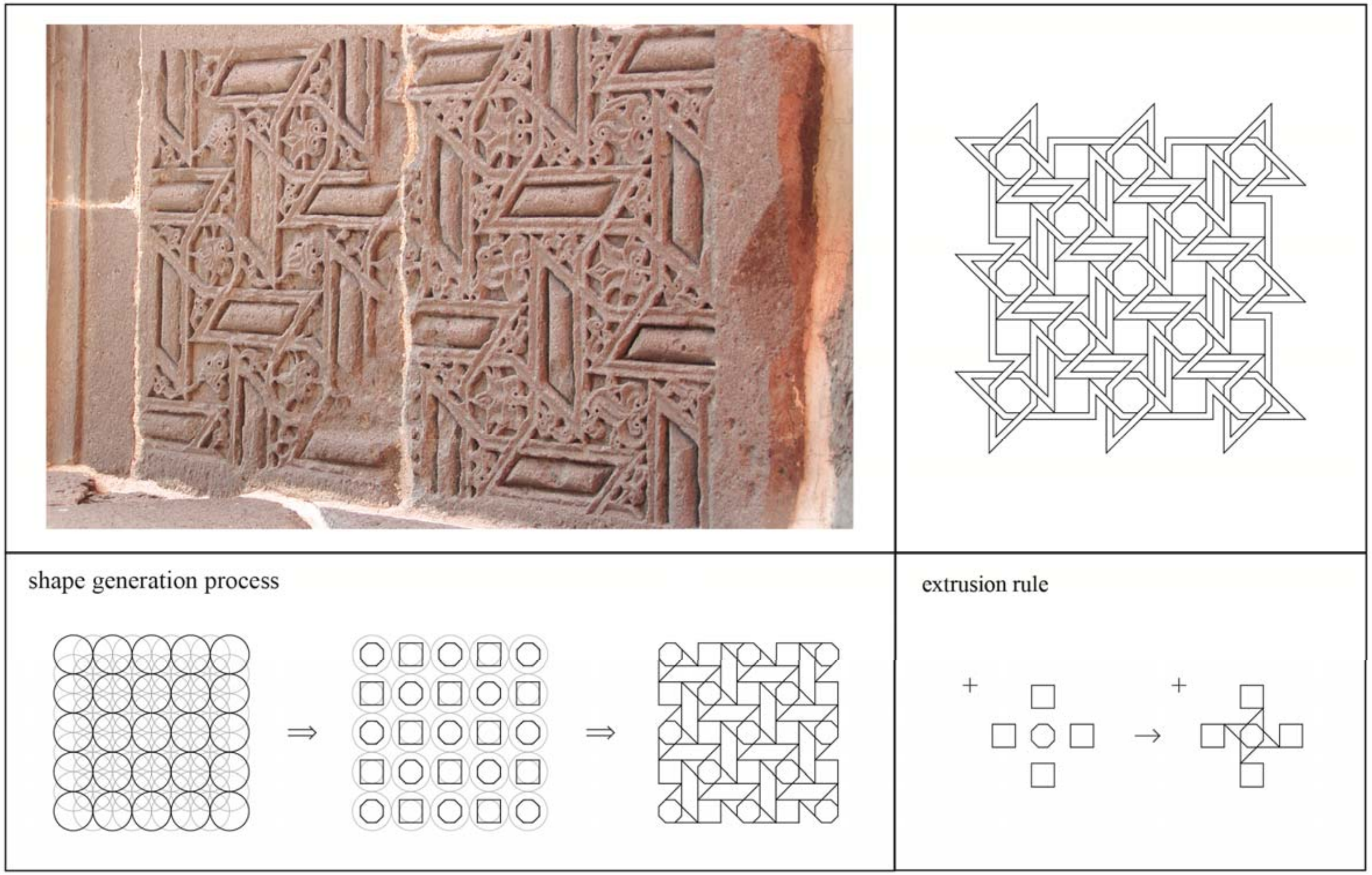

Figure 3.28 : Shape generation process and the extrusion rule of Pattern XVI.

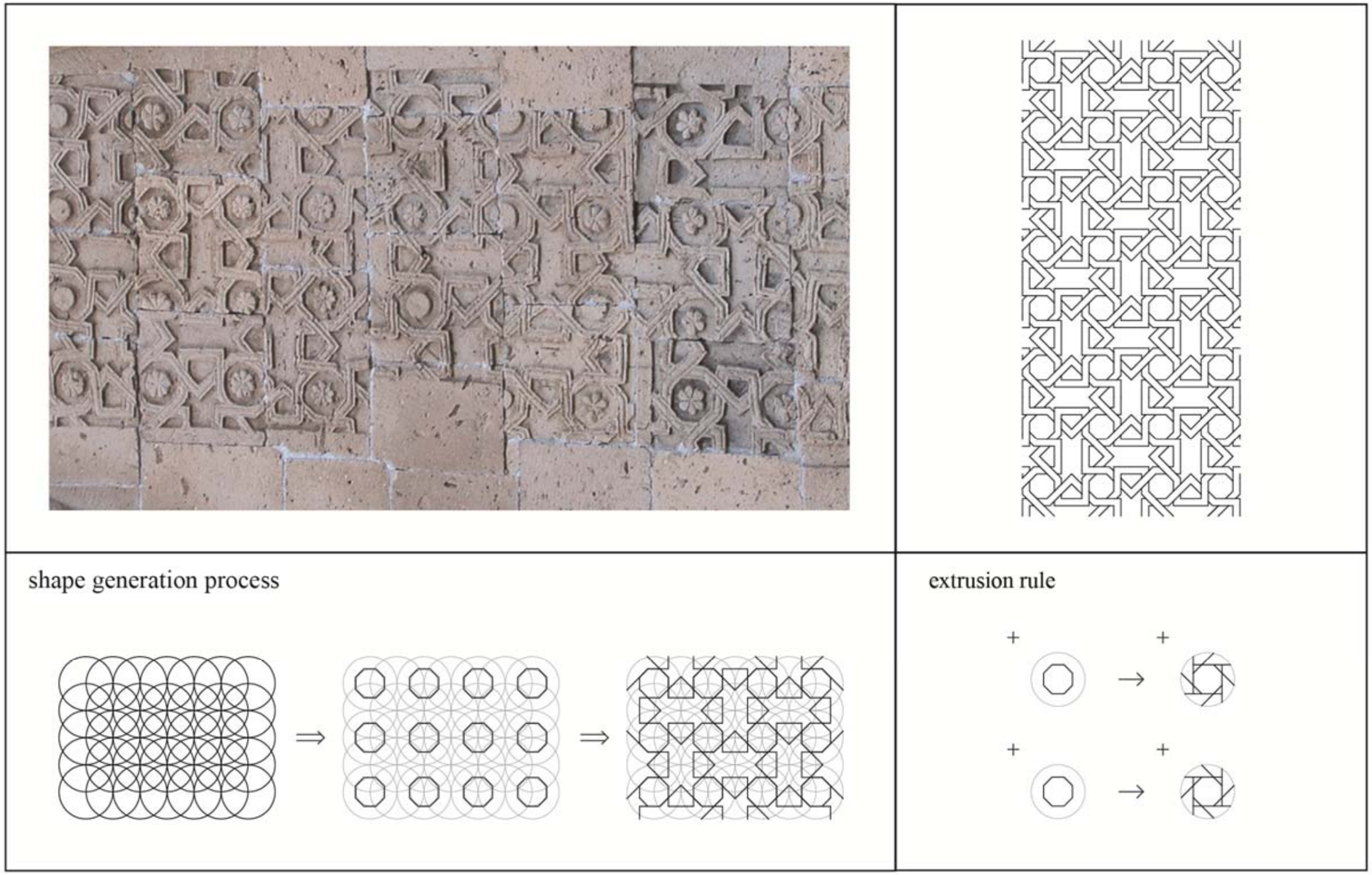

Figure 3.29 : Shape generation process and the extrusion rules of Pattern XVII.

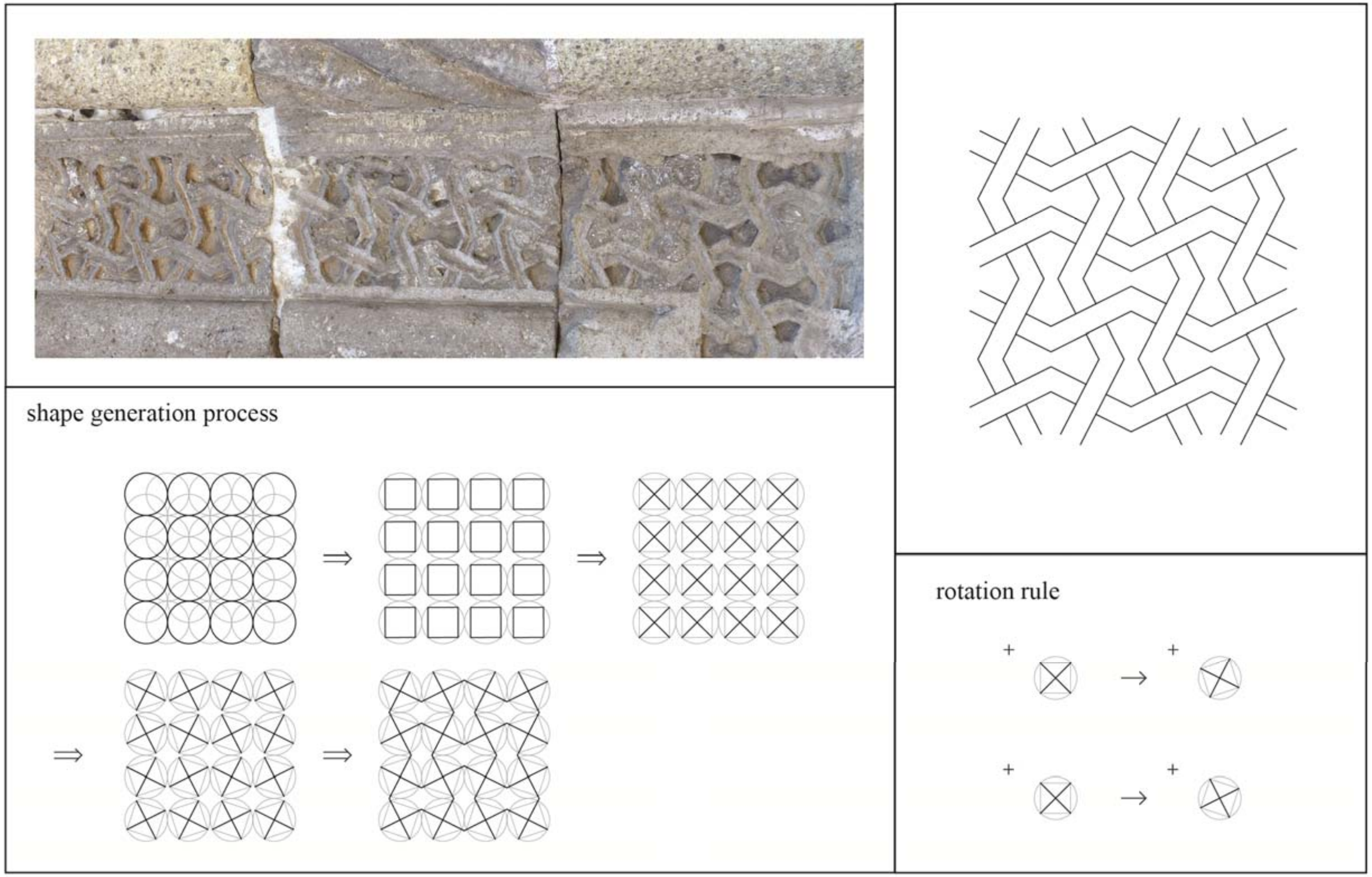

Figure 3.30 : Shape generation process and the rotation rules of Pattern XVIII.

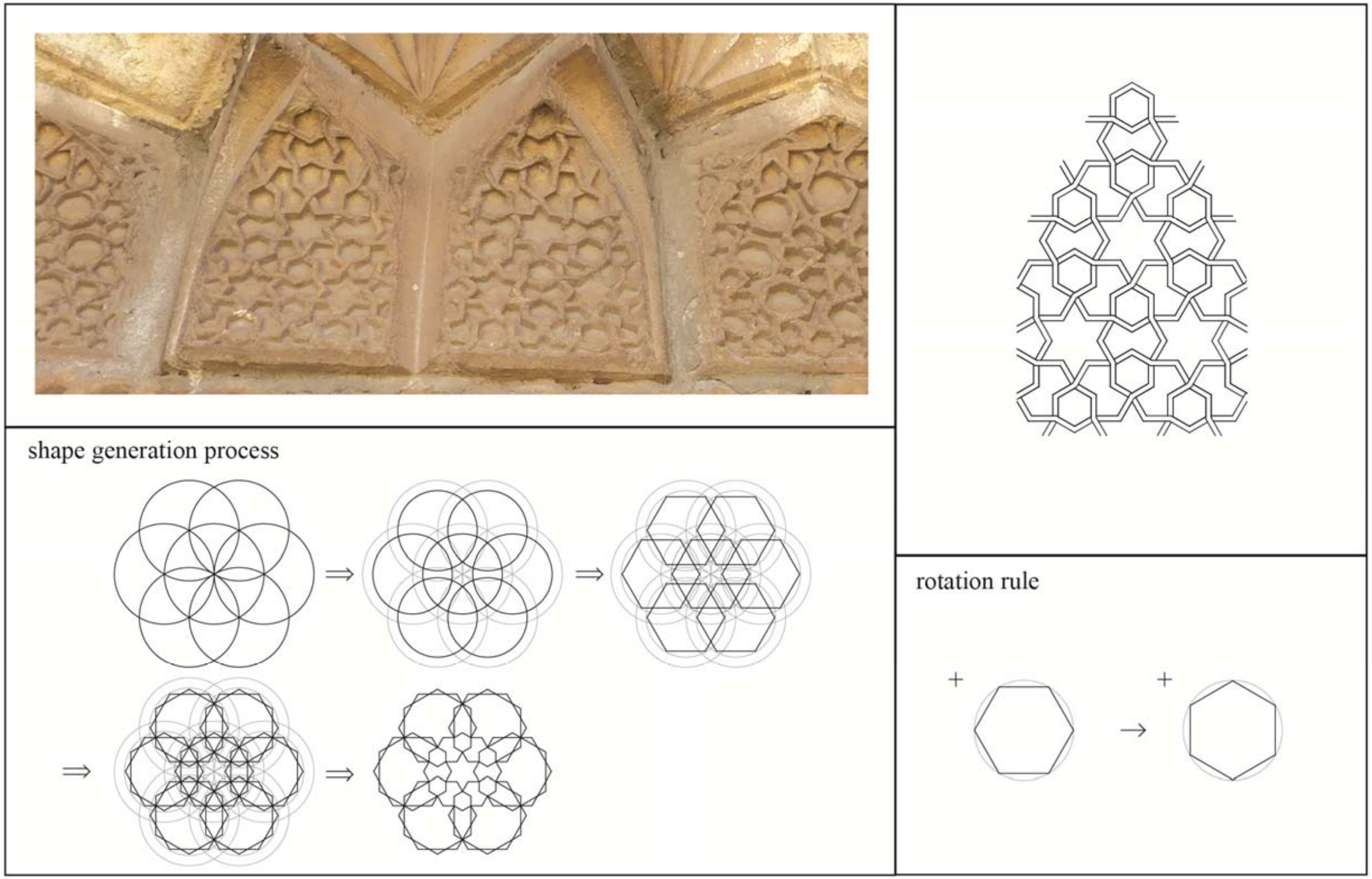

Figure 3.31 : Shape generation process and the rotation rule of Pattern XIV.

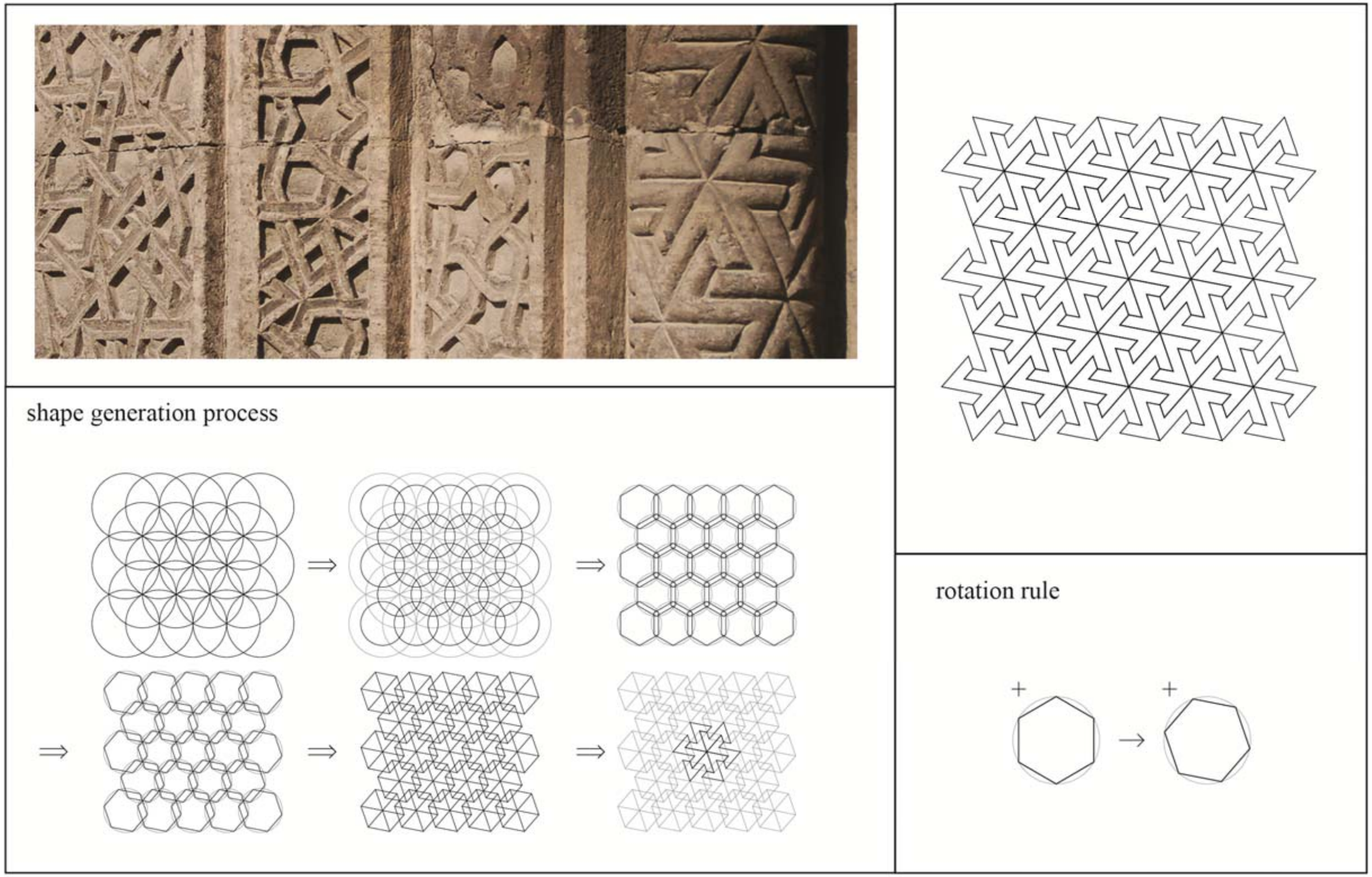

Figure 3.32 : Shape generation process and the rotation rule of Pattern XV.

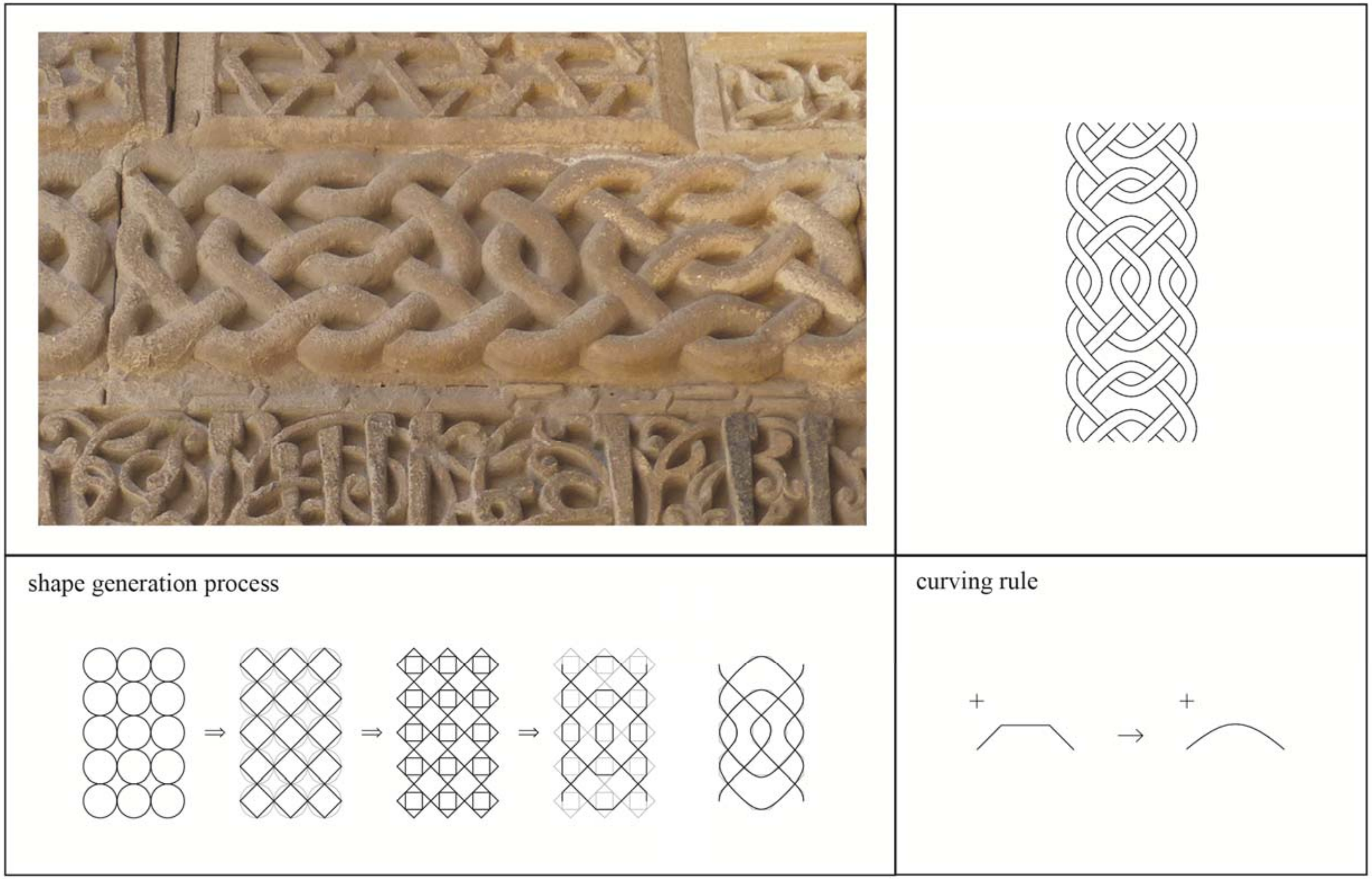

Figure 3.33 : Shape generation process and the curving rule of Pattern XVI.

4. MAKING RULES FOR GENERATING AND PRODUCING SELJUK GEOMETRIC PATTERNS

This chapter will present the analysis of pattern generations that can be derived from the variations in the making process of stone carved Seljuk geometric patterns. Stone carving is a complex process that involves many constituents. Carving geometric ornaments is essentially the transformation of two-dimensional drawings into a three-dimensional pattern as a result of a subtractive process. The resulted material shape is related to the properties of the material, the cutting tools and all the actions that are done with them. Thus, in this case, the pattern generation is the result of material transformations.

Material transformations differ from visual shape transformations. The material transformations analyzed in this study are the ones that are related with the ideation process of the pattern formation. These transformations are constitutive and generate variations as the shape generations do in the previous chapter. By all means, almost each repetition of an application during the making process of a pattern may result in different material outcomes unintentionally. Controlling and computing by hand is not an easy practice, especially on huge stone blocks with variable heights and curvatures. For instance, almost any tile on the repeating pattern examples in the previous chapter is not identical to each other, if they are measured precisely. Yet, this differentiation is not related to the design generation. In Gürsoy's (2016) terms, these transformations are making of, instead of making for. On the other hand, the analysis of the first two patterns in the previous chapter shows that even when the layouts are similar, various material patterns may derive from different actions and interactions in the making process. In this context, the layout geometry is the initial shape of the making process. Therefore, firstly, the layout geometry needs to be transferred onto the material surface. The layout geometry can be replaced on the surface by inscribing with tools like compass and straightedge. The process then continues with various steps of actions such as seeing, placing, carving etc. These actions can be formally represented with making rules in order to reveal the formal relations that generate the material shapes.

Making rules are improved version of shape rules that incorporate material information and constraints. One of the main actions in stone carving is the removing of a particular amount of material by percussive movements. Figure 4.1 shows an illustration of a chisel on a stone block. The chisel is held vertically on the construction lines on the stone. Depending on how deep the strike is, the subtraction forms a three-dimensional niche on the surface. The contouring of the surface transforms the two-dimensional surface of the stone block into a three-dimensional geometry. The shape and diameter of the chisel and the cut depth are the basic parameters that affect the final geometry. The cut depth is labeled as d, and the tool width is labeled as w on the illustration. Moreover, stonemasons can choose between applying the carving process on the construction lines or near them with a particular distance. Therefore, the cut distance is one of the parameters that have been examined in this study and is labeled as m on the illustration.

The making rules have been formalized as the transformation from one state to another as the result of a particular activity. The making rules of different activities are named differently such as carving rules, placing rules, seeing rules etc. in order to differentiate the varşous formalism styles required for each activity. For instance, the carving rule represents the vertical and horizontal sections of both states with lines. The carving rule 1, in Figure 4.1, shows the generic rule of the carving on a construction line. The horizontal section is shown above and the vertical section is shown underneath. The section line is represented by a dashed line and works as a shared label for both sections in order to specify the spatial relation in between. In this context, the first rule always starts with the construction line on the surface, and therefore the vertical section is absent and zero. However, its transformed state on the right side of the carving rule obtains a vertical quantity that is represented by a horizontal line, the width of which equals to the width of the tool width or diameter. Thus, for the next steps of carving the shape in the vertical section will be transformed into another shape or just translated as a result of carving deeper. Moreover, as can be seen on the horizontal section of the same rule, the application results in transforming one single line (the construction line) into two parallel lines. In other words, by formalizing the visible boundaries of the material, the formal aspect of carving through a line can be specified as making one line double.

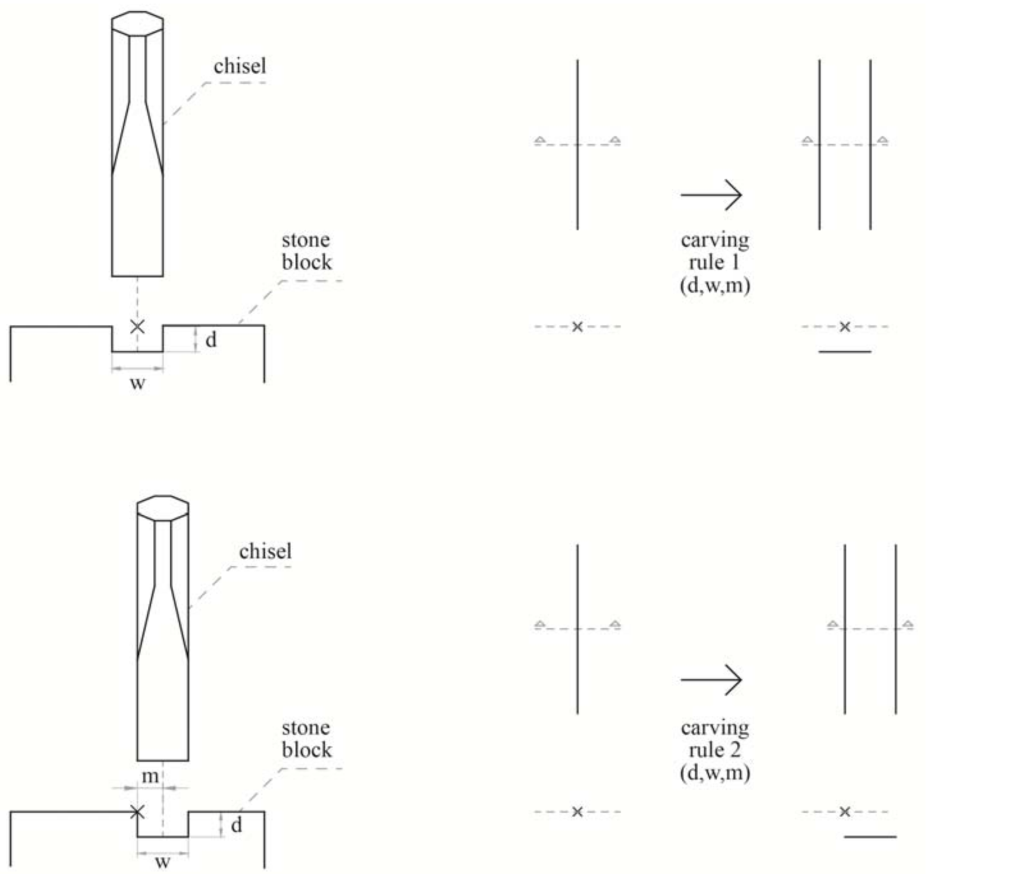

Figure 4.1 : Idealized illustrations and rule formalizations of carving on a line(first row) and along a line (second row).

Moreover, the carving rule 2 shows another alternative rule that differs from the first rule by its distance value (m).The formalization method offers some important insights into the formal relations between the things, activities and the resulting material shapes in the making. Firstly, the impact of all things and activities can be seen on the transformations with the same algebra, which is U_{23} in an algebra U_{ij}. Moreover, formalizing the transformation in both horizontal and vertical planes reveals some formal distinctions of the carving process. For example, if the same tool is used to carve through the same construction line but with more depth, the vertical sections of both rules would differ, whereas the horizontal sections would remain the same. Therefore, in this most straightforward example, one can visually reason about the formal relation between the cut depth and the resulting material shape.

The x marks in the vertical sections label the central axis of the initial construction lines. These marks help to recognize the cut distance value in the application. For example, the relation between the x mark and the line in the resulting vertical sections are different in the carving rule 1 and 2 for the cut distance values vary in these two cases. If there were no x-marks, the distinctions in computing the resulting material shapes would not be recognizable. Thus, such labels in making rules are important as they limit ambiguities in material computations as they do in shape computations.

The scale issue is an important aspect in computational making applications. Unlike abstract shapes, the features of the represented material shapes change as the scale changes. For example, if a material shape is seen closer, more details will be recognizable, and therefore different formal relations might reveal. The formalizations in this study focus on representing the formal aspects that appear important for the generative process of the making process. On the other hand, the consistency between the formalization of the rules and the computations of the same action was taken into consideration in this study.

The making rules so far presented actions that are applied to single lines. On the other hand, the making of Seljuk patterns consists of multiple lines, curves, and surfaces. The computation of the making of multiple lines is not equal to the sum of the computation of each part. For example, the first computation in Figure 4.2 shows that carving two intersecting lines would result in generating four boundaries with corners on the material shape. The second computation in Figure 4.2 shows the carving of two intersecting polygons. In that instance, carving two intersecting polygons generate three closed polygons in the middle. The result of such computations is mostly unpredictable unless formalized or applied. Thus, computations with making rules allow computing emergent material shapes.

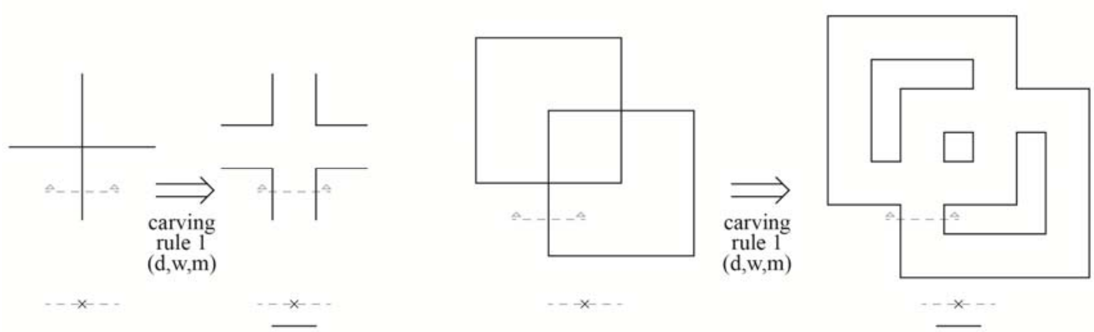

Figure 4.2 : (From left to right) Application of the carving rule 1 to two intersecting lines and polygons.

The shape of the tool is another important parameter in stone carving. The carving rules so far demonstrated the applications with flat chisels. Figure 4.3 shows some of the various chisel types with different shapes. Carving with different shaped-chisels generate different geometries and is, therefore, a very generative tool. Figure 4.4 presents several alternative carving tools that could be applied using differently shaped chisels.

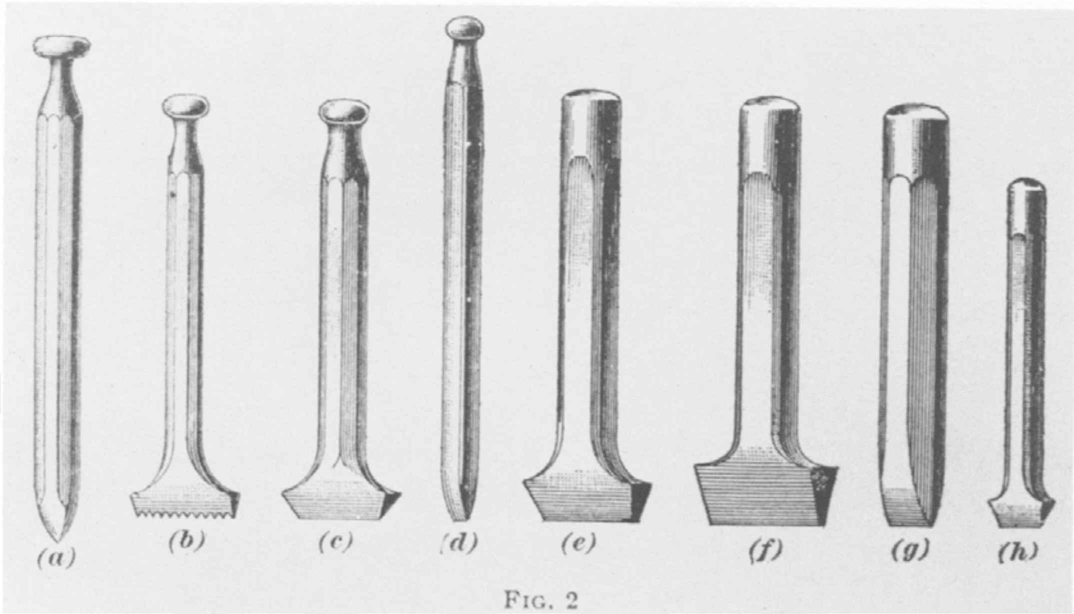

Figure 4.3 : Stonecutting chisels with different shapes (Higgins, 1974).

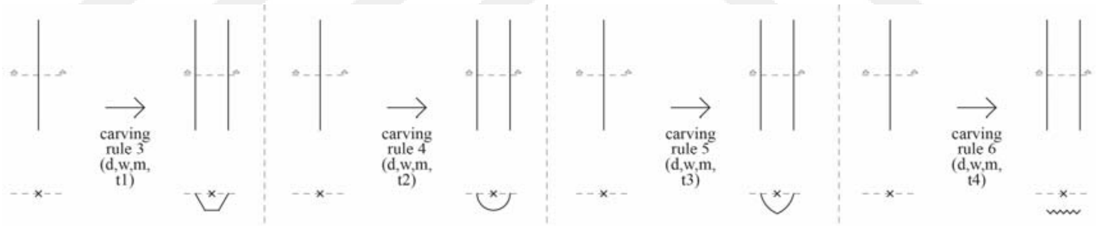

Figure 4.4 : Alternative carving rules using different shaped chisels.

Turning now to the analyzing the existing Seljuk geometric patterns, making rules for each pattern can be derived from the transformation between their layout geometries and physical dimensions. Two existing patterns with the same layout geometry provide a good opportunity to examine variable generations of the making process.

Figure 4.5 represents the study of making rule formalization for each line on Pattern I and II. First, the existing visible boundaries of different levels on the pattern were added to the layout geometry. Then, the transformation from each line to the resulting segmentation on the pattern was presented in the form of a carving rule. As a result of two different carving rule, two different geometries appear on the resulting material shapes. For instance, as can be seen on the images in Figure 4.5, Pattern I resulted in a continuous surface and boundary on the highest level that could be even regarded as a single thick line metaphorically. On the other hand, Pattern II resulted in closed shapes as boundaries of different segments on the surface.

Moreover, the formation in a carving process does not happen in a single step like the casting of a form. In fact, the process includes many sequential percussive movements. Besides, different tools are needed for different forming actions. Therefore there is a sequence in the making of the patterns carved into stone. Usually, the workflow goes from the general to the specific (Wootton et al., 2013). First, there is the rough-shaping of the material, which is the removal of the largest material as possible to achieve the intended form. Second, there is the fine shaping of the material, which constructs the final shape in a more careful manner. Thus, in this particular study, the focus is not the sequencing of the rough and finish stages, instead, the focus is on different steps that might be relevant for generating geometric pattern designs in situ without pre-designing the outcome.

Figure 4.5 : Carving rules for generating Pattern I (first row) and Pattern II (second row) from the same initial shape.

The possible scenarios for generating Pattern I and II can be seen in Figure 4.6. The carving rule 9 and 10 show the generation of Pattern I in two steps. First, in carving rule 9, the carving action is applied parallel to the construction line with a particular distance. Then, in carving rule 10, the curvilinear profile of the surface is generated. In this instance, carving rule 9 is more likely to apply at first for practical purposes. First, the carving rule 9 enables to generate the boundaries that are later used in carving rule 10. Second, removing adjacent materials before working on a surface curvature might be easier for hands-on applications. Thirdly, the directions of the toolpaths on the material surface also need to be different. The carving rule 9 can be applied following the construction line, i. e. a vertical direction. However, the carving rule 10 can be applied by horizontally moving a tool with a smaller diameter in order to trace the curvature of the resulted shape. Moreover, the carving rule 11 and 12 represents the generation of the Pattern II in two steps. The carving rule 11 shows the carving through the construction line on both sides with a particular distance. The v-profile on the existing pattern indicates the use of a V-shaped chisel. In the next step, the carving rule 12 shows the curving of the surface similar to the carving rule 10, but with a different curvature. By all means, this sequencing represents only one of the possible scenarios. More scenarios can be explored through hands-on experiments and discussed in further studies.

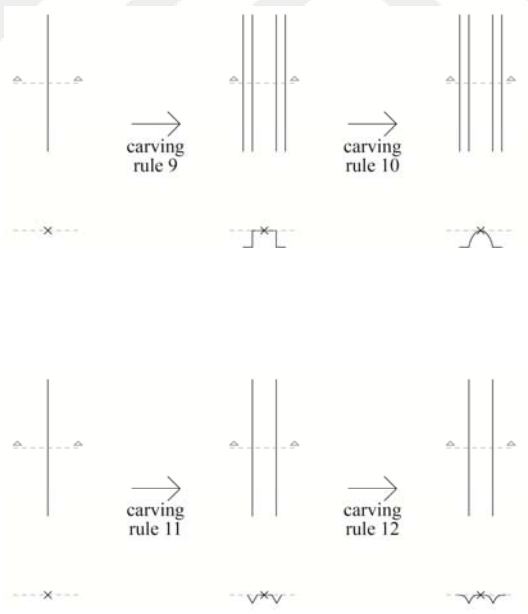

Figure 4.6 : Carving rules for the intermediate transformations of Pattern I (first row) and Pattern II (second row).

The making rules so far presented the applications on single lines that are parts of the layout geometries. Moreover, some applications in the carving process require a redefinition of the boundaries during the practice. For example, the carving of a region cannot be represented with lines, instead, the region should be redefined as a plane with linear boundaries. In fact, there are infinite ways to define discrete parts depending on how they are seen, as it is in the shape computation (Stiny, 2006). Accordingly, eyes can pick new shapes that are embedded in a shape. If we now return to the craft practice of a stonemason, infinite parts can be picked on a geometric pattern to apply specific carving actions. Figure 4.7 represents the definition rule and the carving rule of a surface region as an example. The definition rules and the carving rules are sequential as well. Therefore this study argues that integrating seeing in the making process enhances our understanding of the generative process of making geometric patterns.

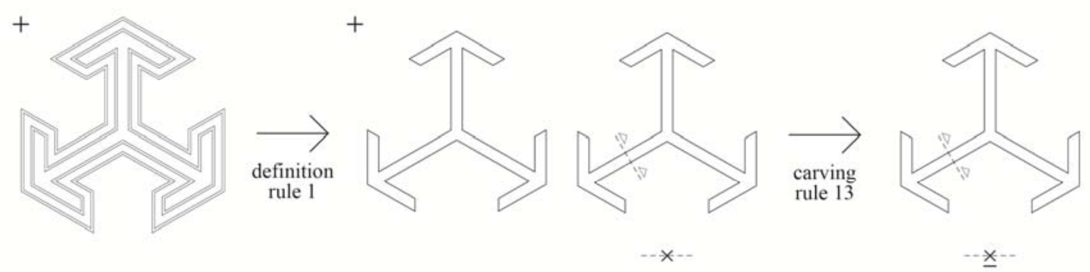

Figure 4.7 : Rules for defining the boundary (left) and carving inside a closed shape (right).

The generative making process of existing patterns can be analyzed as a sequence of definitİon and carving computations. Figure 4.8 shows the six rules for generating Pattern I in a sequential order. The result of this case study on one pattern shows that the generation process of carved geometric patterns is related to the parameters in the carving activities such as distance, depth, tool shape and tool diameter, as well as how the boundaries of these activities are defined on a material pattern.

Moreover, this analysis relies on the unfolded state of the pattern. However, the surface geometry that the pattern is placed on is another relevant issue for two reasons. Firstly, the computation of the making process starts with the application of a layout geometry on a stone surface with particular tools. Thus, analyzing the placement process as a making computation can reveal formal relations between the surface geometry, geometric pattern, material, and tools. Secondly, the placement process may have an impact on the formation of the pattern geometry and therefore be generative.

Overall, the first case study on Pattern I and II suggests that there are three generative stages in the making of stone carved geometric patterns, which are the placement on a stone surface, the definition of the parts and the carving. The rest of this chapter examines how different patterns may generate at each stage, in a sequential order.

4.1 Placing on a Stone Surface

Seljuk geometric patterns were applied on different types of surfaces with positive, zero or negative curvatures. For example, some patterns have been placed inside domes and muqarnas units, whereas some patterns cover columns and hemispherical surfaces on monumental building façades. Craftsmen preserved the complex rotational design symmetries on complex curvatures during the hands-on applications. Visually and manually computing these symmetrical placements require either the knowledge of trigonometry or the know-how of simple drawing tools and techniques.

One of the argued advantages of the circular grid method is the possibility to apply geometric patterns on uneven surfaces (Özkar, 2014). The circular grids can be constructed by using compass, straightedge or different tools. Therefore, in a case study[1], possible scenarios for placing three of the existing stone carved patterns on different types of surfaces have been examined.

The investigations focused on different types of applications using different tools and their relation with the surface geometry and applied design geometries. The case study comprised the examination of three different tool methodologies on each surface types, which are compass, rope, and paper. Each tool has its own formal espects regarding their movement capacities and limitations. Therefore, the tools have been assigned to specific formal algebras for the computation of the making process.

The first pattern is located on the cylindrical surface of an engaged column at the entrance of the Tomb of Mama Hatun in Erzincan. Since the cylindrical surface is developable, the folded and unfolded states of the pattern layout are identical.

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[1] This case study is published in Bridges Finland 2016 Mathematics, Music, Art, Architecture, Education, Culture Conference Proceedings, written in collaboration with Mine Özkar. The corresponding section in this chapter is largely from this publication, titled “Geometric Patterns as Material Things: The Making of Seljuk Patterns on Curved Surfaces” (Hamzaoglu & Özkar, 2016b).

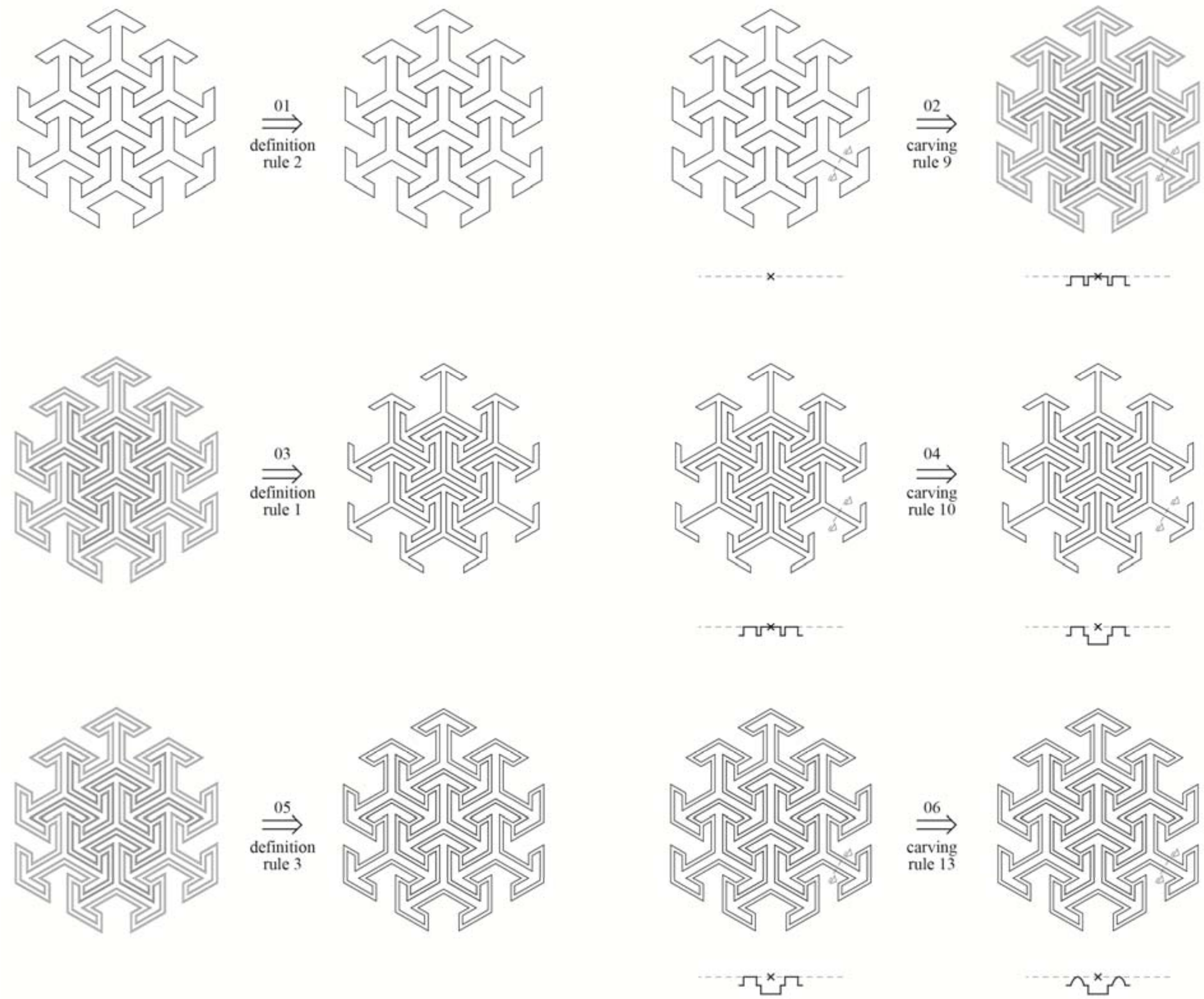

Figure 4.8 : Computations of the making of Pattern I.

The pattern layout can be constructed from a circular grid with a six-fold rotational symmetry as shown in Figure 4.9. The lightly drawn circles indicate the intermediary construction circles, that are used for generating the next circles. In the first alternative application, the circular grid is drawn on a sheet of paper or tissue that has a length equal to the circumference of the cylindrical surface. The radii of circles are equal and follow a linear and uniform repetition on the rectangular sheet. the ratio of the radius of a grid circle to the radius of the cylinder is $\pi/3$. The paper wraps around the cylinder and acts as the template as its dark lines press onto the stone and mark it. Figure 4.10 shows the placement rule of this application. Afterward, the circular grid and the additional geometric shapes can be inscribed onto the surface with a chisel.

In terms of formal algebras, a paper is a plane when placed on a planar surface and therefore can be formalized with the algebra U_{22}. Yet, in this case, wrapping of paper is computed in the algebra U_{23}.

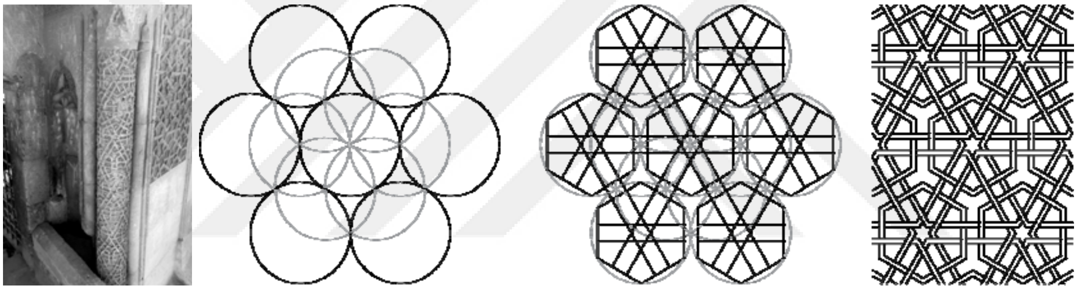

Figure 4.9 : (Left) Photograph of the pattern on the engaged column at the entrance of Tomb of Mama Hatun. (Right) A circular grid and the pattern constructed from it.

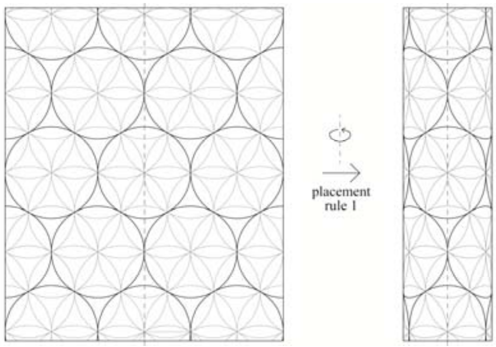

Figure 4.10 : Placement rule showing how the paper wraps around the cylinder.

The second alternative application is based on using a rope for placing the circular grid on a cylindrical surface. Figure 4.11 shows the sequential drawings and photos from the experiment of this alternative. The circles are drawn one by one based on the intersection points of te former circles. One hand locates the center of the next circle by pressing the end of a rope from the top and the other hand holds a marking tool and stretches the rope so that it follows the curved surface. The length of the rope is equal to the radius of each circle drawn. The hand with the marking tool then turns around the center to draw o full circle. As a result, the circles have the same radii as on the planar surface of the paper. Besides, it is important that the hand that locates the center point should be pressing the rope from the top and not cover its sides so that the length of the rope won't change during the rotation of the rope.

In this application, craftsmen's arms are used as a flexible and organic compass that can draw circles on curved surfaces with the help of a piece of rope for specifying the radius. Considering the dimensions of the tool, the rope can be computed in the algebra U_{13}. Yet, the movement capacity of the craftsmen's arms limits the maximum dimension that the pattern can be generated of.

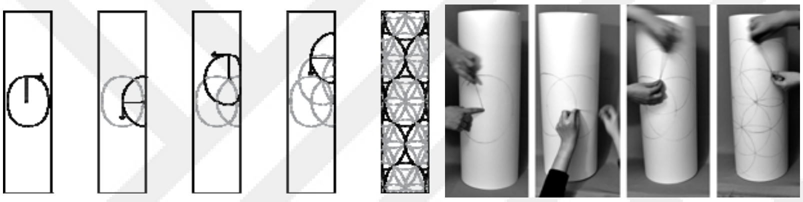

Figure 4.11 : (above) Drawings that show how each circle is drawn on the cylinder using a rope. (below) Photos showing the process.

The second pattern is located on a curved squinch on the same monument. A squinch is a typical muqarnas unit, that has been used in Islamic architecture for covering vaults. Figure 4.12 shows that the pattern can be constructed from a circular grid with a six-fold rotational symmetry.

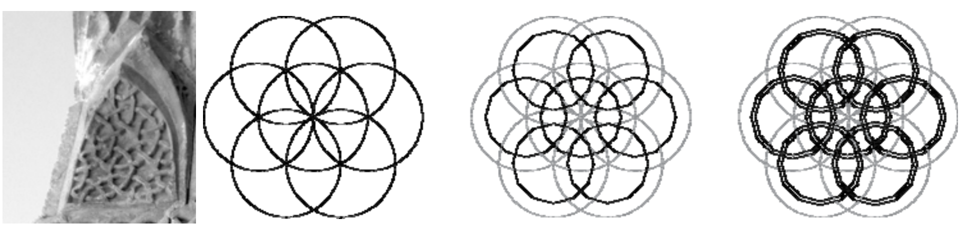

Figure 4.12 : (Left) Photograph of the pattern on a curved squinch at the entrance of the Tomb of Mama Hatun. (Right) A circular grid and the pattern constructed from it.

The curved muqarnas units are constructed according to the triangular cell structure of their plane projections, in which the angle between two vertical edges meeting at is usually 90°, 45° or 135° (Dold-Samplonius, 1992/3). The cells are then raised and the curved surfaces expand between the edge of the lower cell and the corner of the cell above. Figure 4.13 illustrates the developable surface inside this particular squinch.

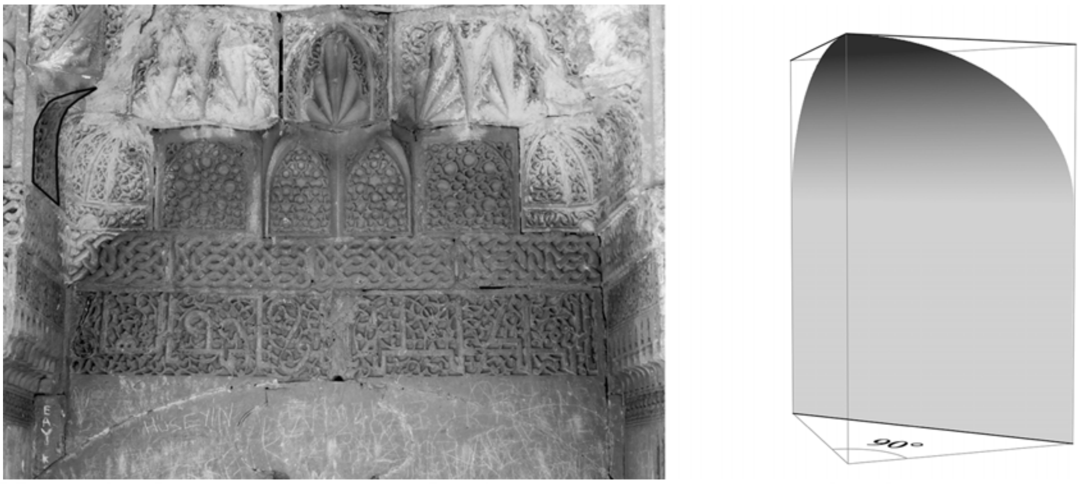

Figure 4.13 : (Left) Photograph shows the location of the curved squinch. (Right) The geometric model of the squinch that develops upwards then curves.

In the first alternative, the circles can be transferred on the surface by using paper as a tool again. The circular grid pattern is drawn on a paper and the paper is bent on the curved squinch. Figure 4.14 shows the placement rule for the application. In this case, bending of paper is computed in the algebra U_{23}.

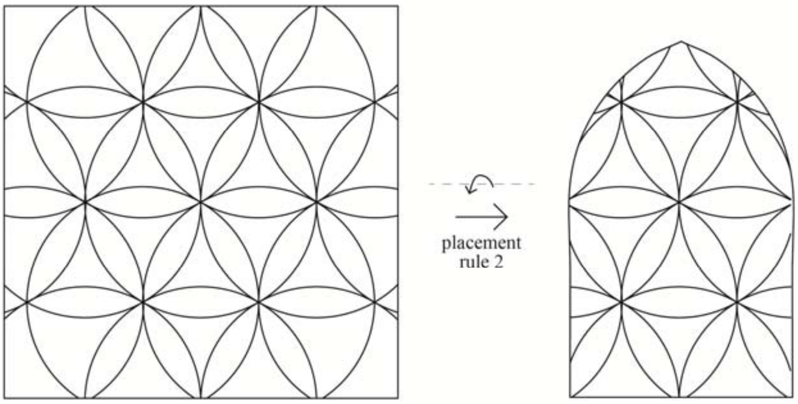

Figure 4.14 : Placement rule showing how the paper is bended on the squinch surface.

However, if the bottom edge of the squinch were curved, as it sometimes is in muqarnases, the surface then would not be developable and the paper template application would require extra steps such as physically modeling the unfolded state of the surface as interrupted parts. On the other hand, the material of the paper would also change the placement of a geometry on an undevelopable surface. For example, if the material is flexible, the paper can be placed on an undevelopable curved surface. Yet, in this case, the layout geometry that was drawn at the unfolded state would also be bent when placed on the surface. Therefore, a different design layout would be generated.

In the second alternative, shown in Figure 4.15, circles can be drawn by using a rope. This method is similar to the second alternative of the first pattern. However, in this case, the geodesic distances between the points on the circle and its center because of the concavity of the surface. Therefore, the resulting shape is not a perfect circle. This application could be used for this pattern since the main part of the geometry is located on the flat area of the surface and the curved area is relatively small. As a result, the distortion of the geometry is hard to recognize at this scale.

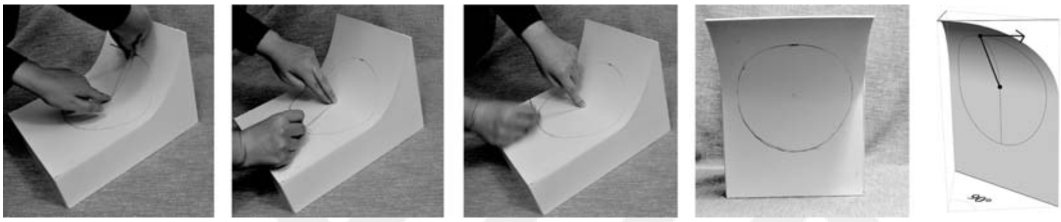

Figure 4.15 : (Left) Photographs show the drawing process using rope. (Far right) Drawing illustrates the stretched rope and the geodesic distance between the shape and the center.

The last example of this case study is a pattern on a hemisphere-shaped stone surface from the entrance of the late 13th century Buruciye Madrasah in Sivas. Figure 4.16 shows the image and the drawing of the pattern.

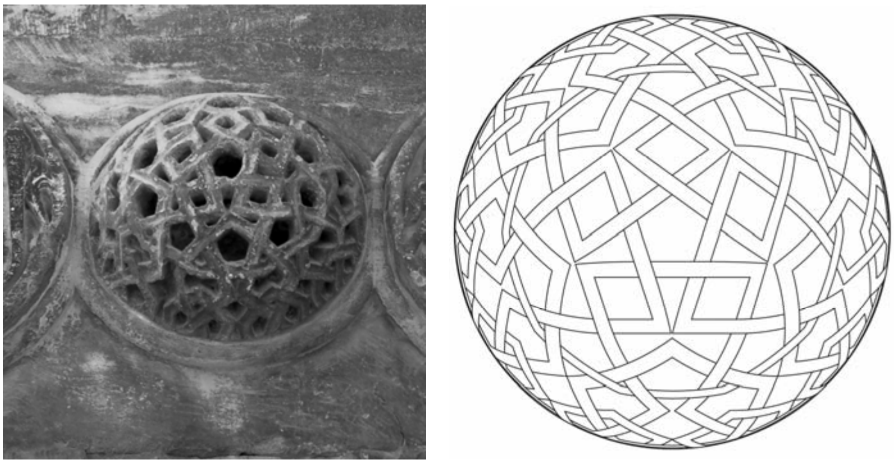

Figure 4.16 : Photograph (Left) and drawing (Right) of the pattern in Sivas.

The surface geometry of the pattern is not developable and therefore the folded and unfolded states are not identical. Figure 4.17 shows the unfolded state of the circular base and the pattern constructed from it. The circles are not in contact in the unfolded state as they are when folded on the hemispherical surface. Therefore the geometry can only be constructed on the stone surface with the circular grid method.

The pattern is comprised of interlocking geometric shapes of various sizes. The illustration of the unfolded state in Figure 4.17 reveals the layout of the geometric shapes on the pattern. At the center, there is one central pentagon that is divided into five rhombuses, five triangles, and a pentagon. The triangles and a pentagon constitute a star. These five triangles may also be perceived as five trapezoids because their lines are thickened up. Then there are ten other pentagons of the same size around the central polygon. Then, there is a single decagon that goes through the centers of these pentagons. Finally, towards the outer rims, there are ten additional smaller pentagons around the decagon that are interlocked with the bigger pentagons.

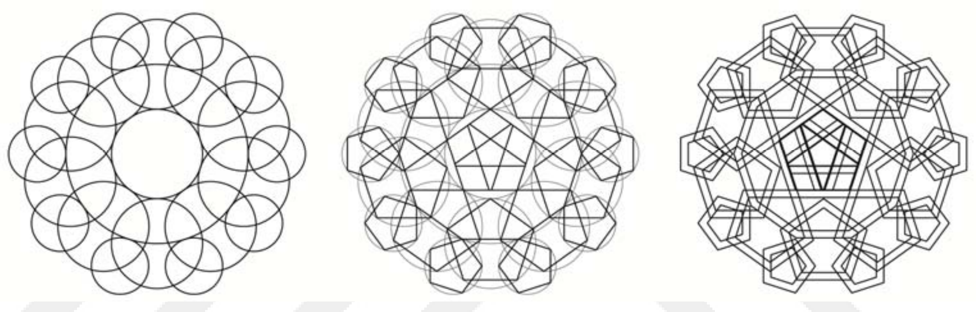

Figure 4.17 : The circular base and the polygons for constructing the pattern.

Moreover, the circular base includes circles of three different sizes. The diameter of the smallest circle is equal to the radius of the intermediate circle and the diameter of the intermediate circle is equal to the radius of the largest circle. Figure 4.18 shows the application of the circular base on the surface with the rope-compass method. The process of application starts with drawing two concentric circles with varying radii at the top of the hemisphere. The smaller one of these, with a diameter ½ the radius of the hemisphere, is for inscribing the pentagon and the bigger one is for the decagon. Then another circle that has the same radius equal to the small one is drawn around one random point on the big circle. The process continues with drawing nine more circles in a way that each time the next circle intersects at the center of the latest drawn circle. Finally, the smallest circles on the grid can be drawn around the intersection points of the ten existing circles around the center. In this way, differently sized polygons can be adjusted on a spherical surface using simple tools. The same method can also be applied by using a regular compass instead of rope-compass in this particular case of the hemispherical surface.

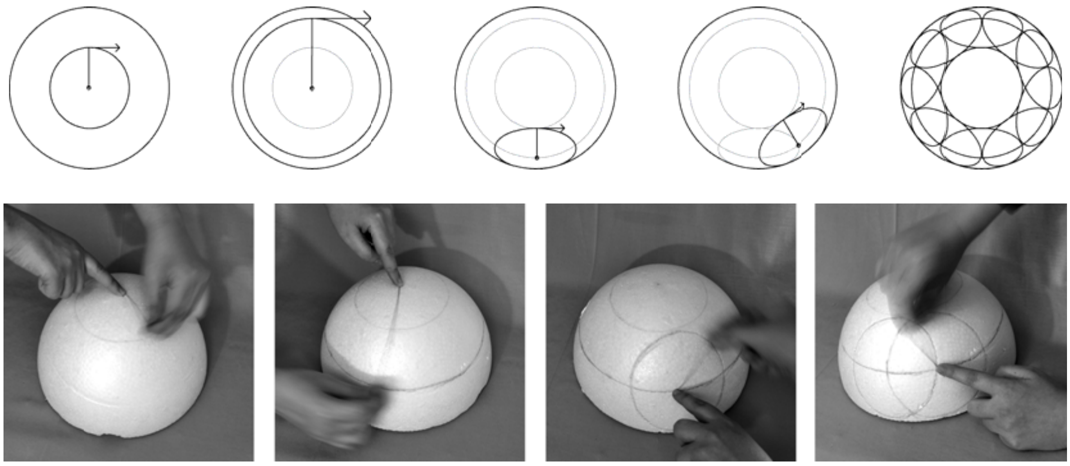

Figure 4.18 : Each circle is drawn on the hemisphere using a rope or a compass.

Overall, this case study examined the first generative aspect of placing the pattern on different surface geometries, which is the formal relation between the tools, actions and surface geometries. The results indicate that the formal algebras in which a tool can be computed can vary depending on the dimensional possibilities and limitations of the action that is done with a particular tool. Moreover, the relation between the surface geometry and the formal algebra of the tool has an impact on the variations of the pattern design.

Furthermore, there are examples of existing Seljuk geometric patterns, on which patterns appear to be transformed due to placing them on uneven or arched surface geometries. For example, the differentiation of tiles on the arch at the courtyard of Mahperi Huand Hatun Complex in Kayseri can be examined as transformations. Figure 4.19 shows the location and the images of two different tiles on the arched ornament, together with the rule of the pattern transformation as a placement rule. The rule shows that the square shaped carved area is identical in two cases, yet the outer boundaries are different-shaped. The reason for this differentiation might be that the patterns were fit accurately inside the boundaries of the stone tiles they are applied on. Therefore, the patterns appear to be carved in situ after the arch was built.

Figure 4.19 : Deformation of the geometry placed on different tiles of the arch and its placement rule formalization.

Placing a pattern on a surface is a comprised of multiple sequential stages. The possible processes for generating pattern B1 and B2 have been formalized as a series of computations (Figure 4.20). The first row shows the computation of the pattern B1 on the square-shaped tile, whereas the second row shows the computation of the pattern B2 on an acute angled archstone at the top of the arch. Both processes start with drawing one of the circles from the grid of nine circles with an eight-fold rotational symmetry. The layout is comprised of three circles in each row and in each column. The stone tile on which the pattern B1 was placed has a fixed width. Therefore, the first circle can be placed at the center of the tile and the other eight identical circles would fit inside a square. A horizontal line can be drawn from the midpoints of opposed edges. The first circle can be applied by using a simple compass. The radius of the circle needs to be one-quarter of the horizontal line. However, the placement of pattern B2 needs to start from drawing one of the circles from the bottom row of the grid, where the stone tile has the minimum width. The other eight identical circles of the grid and the geometric shapes can then be applied to the tile. As a result, the pattern B2 will be smaller than the tile that it is placed on. Therefore, at the final stage, some of the geometric shapes of the pattern can be merged with the boundaries of the tile by the craftsmen and some parts can be erased for generating the layout geometry on the tile. This example shows that the shape formations vary according to the surface geometry of the pattern.

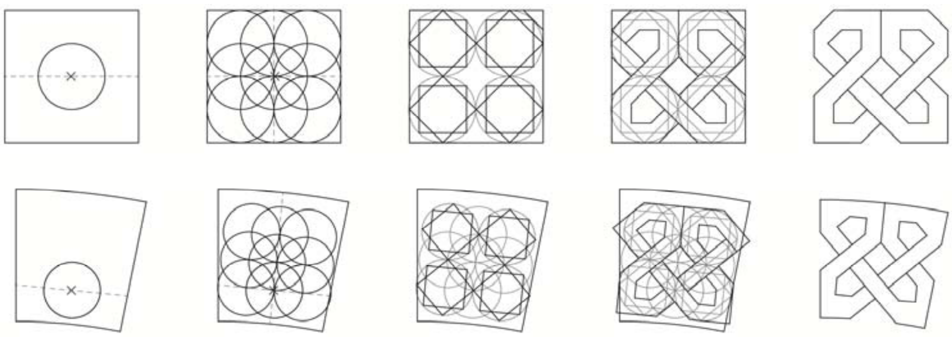

Figure 4.20 : Sequential stages of placing the pattern on a square-shaped and an acute angled archstone.

Another example for the transformation of geometric shapes by placing on a specific surface has been found at the portal of the hospital in Amasya. A part of a complex geometric pattern was placed on a hemispherical surface. The difference of this pattern from the pattern on the hemispherical surface in Sivas is that in this case, the pattern continues and expands between the flat area and the hemispherical surface.

Therefore, the intermediary geometries were transformed by stretching them towards the reference points of the pattern on the flat area. Figure 4.21 shows the images and the drawing of the pattern, together with the illustration of possible three stages of the placement computation. Firstly, the spherical base is constructed. Secondly, the pattern layout has been applied on the surface. Finally, the corners of quadrangular shapes on the flat area have been moved to the corners of the quadrangular shapes on the hemispherical area.

The last two pattern examples show that different geometric patterns can be generated by placing the patterns on curved or arched surfaces. Therefore, the placement process can be integrated within the design ideation process of a computational geometric pattern design considering the features of the surface geometry such as curvature value, dimension, and boundary shape.

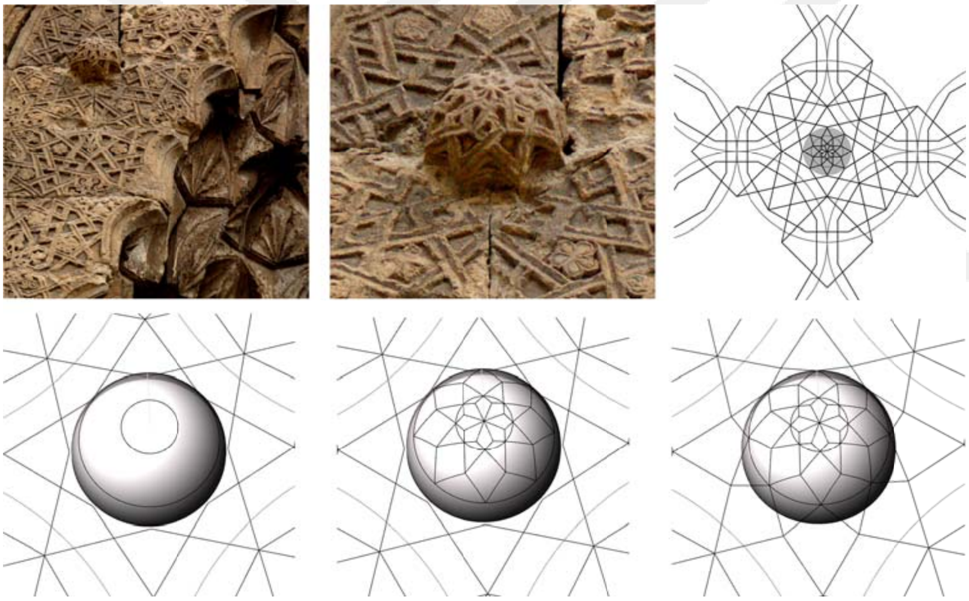

Figure 4.21 : Transformation of a 2D pattern by extending towards a 3D pattern placed on a hemisphere.

4.2 Defining Parts

The second part of the making process examined in this study is the definition of the parts of the patterns to apply the carving action. This intermediary process is a visual activity. As in the shape computation (Stiny, 2006), infinite parts can be defined on a pattern depending on how it is seen. The shapes that are seen on the layout geometries fuse and the definition of the parts do not need to be related with which particular shapes were considered to geometrically construct the whole layout. For example, two pattern examples in Figure 4.22 represent this differentiation. The pattern above is comprised of multiple lines within a four-fold rotational symmetry, that can be generated from subtracting some parts of the square shapes. The definition Rule 1 indicates that vertically repeating planes were defined to be carved by their boundaries on this particular pattern. The carving Rule 1 formalizes the carving of the planes and how the boundaries change vertically and horizontally after carving. Additionally, the definition Rule 2 indicates the definition of other planar parts that are then carved with the carving Rule 2.

Repeating geometric shapes on a pattern may form unintended geometric shapes as they fuse together. For instance, the definition Rule 3 in Figure 4.22 represents the definition of rhomboid shapes from a pattern that may have been constructed by extending octagon shapes. The rhomboid shapes were embedded in the pattern. Therefore, the definition of the parts has an impact on the resulting stone carved geometric pattern.

The layout geometries of the example patterns examined in this study are planar entities. Therefore, parts of them can be defined as points, lines or planes. A plane can be defined by its boundary, which is a line. The definition is related to the carving process as well. For example, if a point is defined, then the carving action will be drilling. If the toolpath is defined as a line, then the action will be engraving or cutting. If the toolpath is defined as a plane, then the toolpath will comprise multiple lines that cover that plane. Hence, by defining different parts to apply different actions variable patterns can be generated from a single initial layout shape.

4.3 Carving

In this study, the carving process has been analyzed in terms of carving distance, carving depth, tool shape and tool diameter. This section examines variable pattern generations by changing each parameter particularly.

4.3.1 Distance

The chisel can be located at the sides of a construction line with a particular distance. As a result, the boundaries of the carved geometry will differ from the initial layout geometry depending on the distance between the chisel and the construction line. When formalizing the process as a series of computations, the central axis of the construction line can be labeled with x-marks for differentiating various distance values, as shown in Figure 4.23.

Two pattern examples in Figure 4.23 show that the distance parameter can be also used to articulate surface forms and textures. For example, the pattern above appears to be carved on both sides with the same distance. As a result, cylindrical parts are generated along the construction lines on the stone surface. Similarly, the pattern below exemplifies the case of articulating a surface by engraving lines. In this case, the construction lines are transformed into a cluster of three engraved linear paths. Together, these examples highlight that different distance values can be used for generating various patterns.

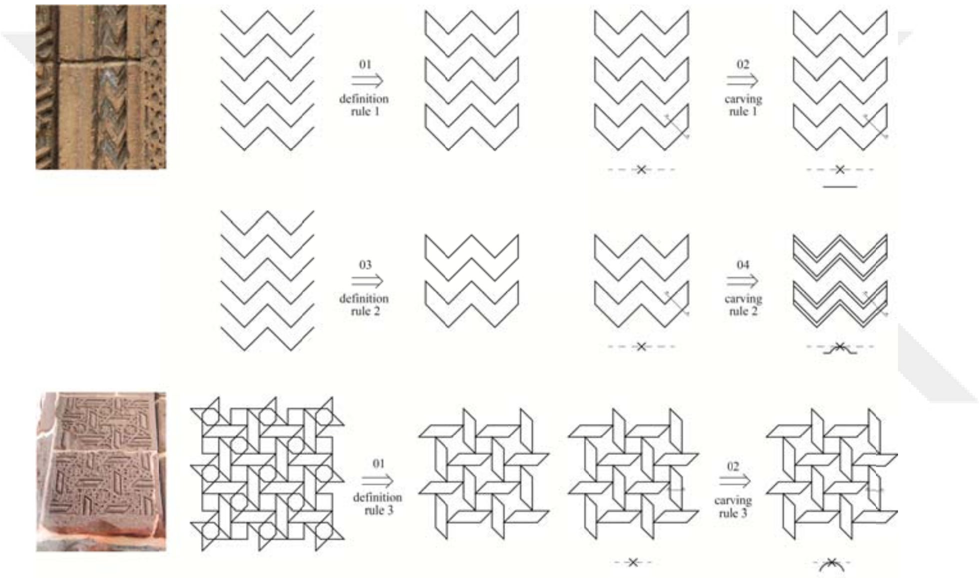

Figure 4.22 : Two examples of pattern variations by defining the parts and applying the carving rules accordingly.

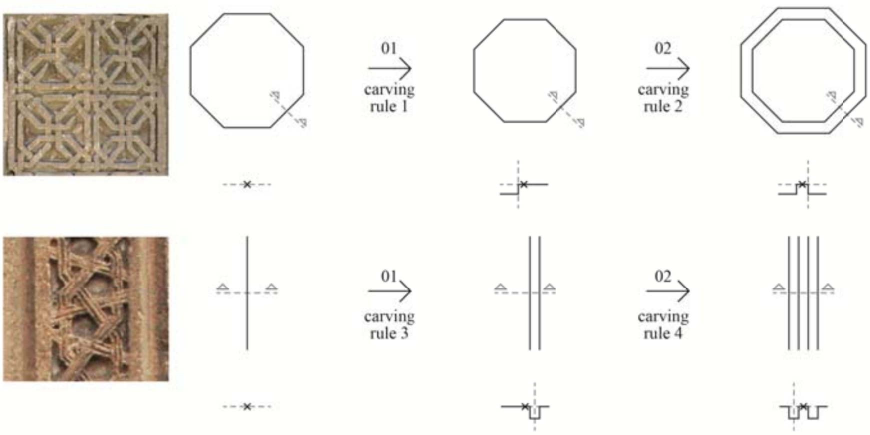

Figure 4.23 : Two examples of pattern generations by using particular carving distances.

4.3.2 Depth

The depth value of a carving process indicates how deep the chisel will be struck inside the stone blocks. The value can be defined with fixed length or a vertical section line. Some of the existing stone carved patterns have curved profiles, that can be carved by defining a particular curve as the depth value of the surface. For example, the pattern on the left of the first row in Figure 4.24 has been articulated by using varying depth values on particular parts. For example, The hexagon shaped planar parts of the design layout have been carved deeper on the edges, as shown in carving rule 1. As a result, the hexagon-shaped parts of the design layout are separated from their neighboring parts at the surface area and therefore highlight the geometric design by creating shadow effects at their boundaries. Moreover, the example pattern on the right and the other possible carving rules on the second row represent different carving rules to generate various stone carved geometric patterns.

4.3.3 Tool Shape

As mentioned at the beginning of this chapter, differently shaped chisels are can be used during the carving process. Thus, various patterns can be generated by engraving through a construction line with different chisels. Figure 4.25 shows two pattern examples that can be generated by using particular chisels. The pattern on the left can be generated by engraving the construction lines with a sharp-pointed tool, whereas the pattern on the right can be generated with a ball end cutter. Besides smooth shaping, these tools outline particular boundaries on the surface around them. Therefore various geometric patterns can be generated by using different shaped tools on a single initial layout geometry.

4.3.4 Tool Diameter

Similar to the tool shape, tool diameter is another parameter that can be used to generate various geometric patterns in the carving process. The tool diameter specifies the width of the carved area on each point on te surface that the cutting tool was stroke onto. Since geometric patterns are comprised of multiple construction lines, various geometric compositions can be generated by using tools with different diameters on them as the boundaries of carved areas will change on the pattern. Figure 4.26 shows three pattern generations by using three different carving rules. The patterns in the first two rows are generated by engraving the lines with two different tool diameters. The impact of the tool diameter is related to the dimensions of the layout geometry. In this case, different geometric compositions are generated from the same initial layout geometry. In the first pattern, the resulting boundaries of the uncarved areas are connected as one large geometric shape. However, in the second pattern, the resulting boundaries of the uncarved areas are multiple separated geometric shapes.

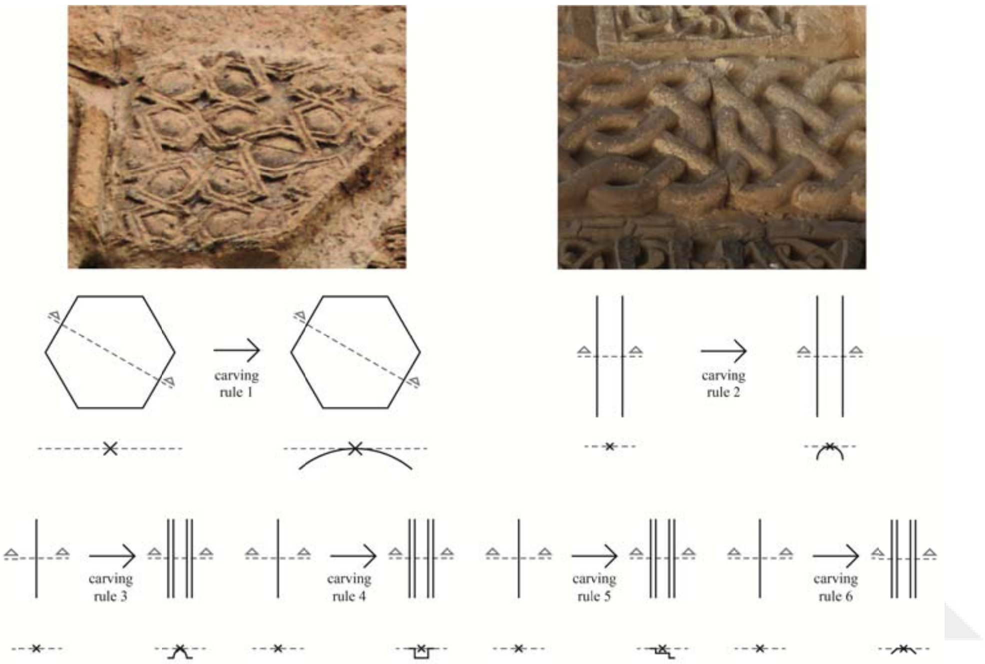

Figure 4.24 : Two examples of pattern generations by different surface curvatures for defining the carving depth (above) and four alternative depth definitions for generating various patterns (below).

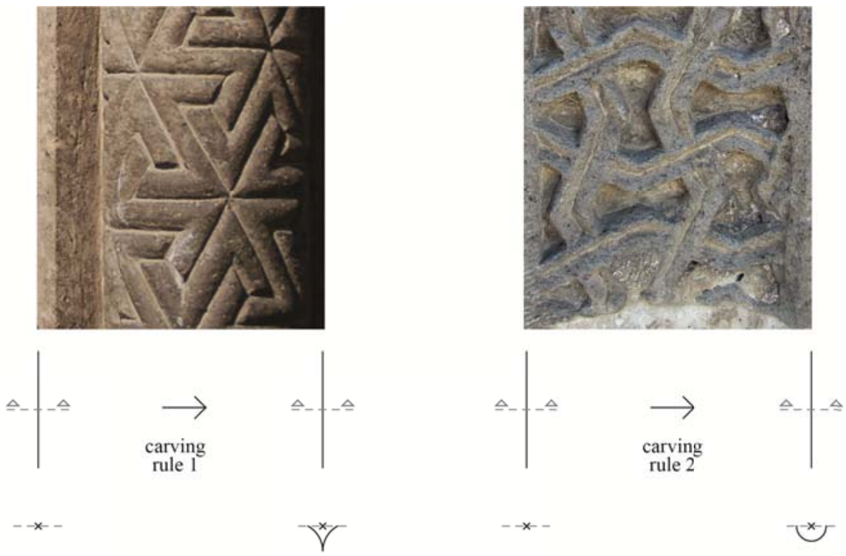

Figure 4.25 : Two examples of pattern generations by using tools with different shapes.

Lastly, the third pattern in Figure 4.26 demonstrates the impact of carving out planar parts of the material surface with a particular tool diameter. The tool can not engage in the parts that are smaller than its diameter. As a result, the corners inside the boundaries of the carved parts are rounded with the same radius as the carving tool. Consequently, the carved areas are separated from each other on the resulting pattern. Together, these examples show that various geometric patterns can be generated depending on the formal relations between the tool diameter, the dimensions of the geometric shapes on the initial layout and the definition of the parts to be carved as lines or planes.

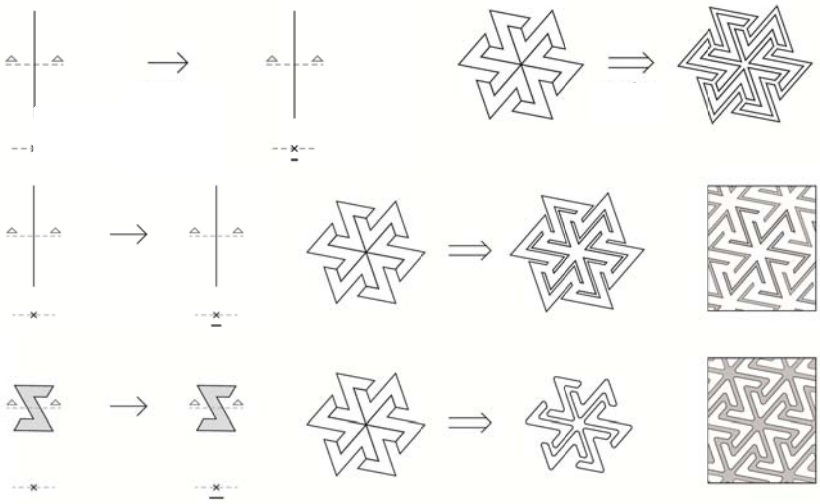

Figure 4.26 : Three examples of pattern generations by using flat-shaped tools with different diameters.

5. APPLICATION OF MAKING RULES IN DIGITAL FABRICATION

Making grammars provide a generative system for analyzing and generating material productions using making rules. In this chapter, experiments on applying making rules with digital fabrication tools for generating new pattern designs will be presented. The case study focuses on generating new patterns by computing the making of a 3-axis CNC milling process since it is a subtractive production method like stone carving. Although the milling toolpaths differ from a stonemason's hand movements, the resulting material shapes are formed by similar making parameters such as tool shape, tool diameter, depth, distance, part definition and surface geometry.

The computation of the making of a 3-axis CNC milling process is controlled by scripts applied by the mechanical system of the machine. The toolpath is defined by the machine language named G-Code. The code consists of alphanumeric characters that define coordinates and particular motions. There are also Computer Aided Manufacturing (CAM) softwares special for each CAM tools that provide simple and quick toolpath generation. These softwares enable users to choose or define the making parameters in visual interfaces based on the digital model of the desired material shapes. Digital models provide reference points for the toolpath coordinates. Yet, the resulting shape of the milling process is computed as a result of a process. Therefore, various material shapes can be obtained from a single digital model by using different milling parameters.

A basic toolpath generation process on a CAM interface usually starts with choosing a digitally modeled geometry. Then, the user decides which machining operation to apply. There are multiple preset operation options such as engraving, pocketing, facing, profiling, horizontal roughing, horizontal finishing, parallel finishing, hole drilling etc. Each operation can be used for different purposes depending on the desired geometry. Moreover, each operation requires particular part definitions. The parts can be defined as points, lines, planes or three-dimensional surfaces on digital models. For instance, in an engraving operation, the tool follows linear or curvilinear paths. Therefore the parts are defined as lines or curves. Figure 5.1 shows the 3-axis CNC Milling machine used throughout this study and a sample engraving process on a circle-shaped path. Some operations such as pocketing require a part definition in form of planar closed shapes like a boundary of a plane shape, so that the inside of the boundary will be defined as a pocket. Some other operations such as parallel finishing require a part definition in form of surfaces. These surfaces can be two- or three-dimensional and the tool will flow along the surfaces by following linear paths that scan the surfaces.

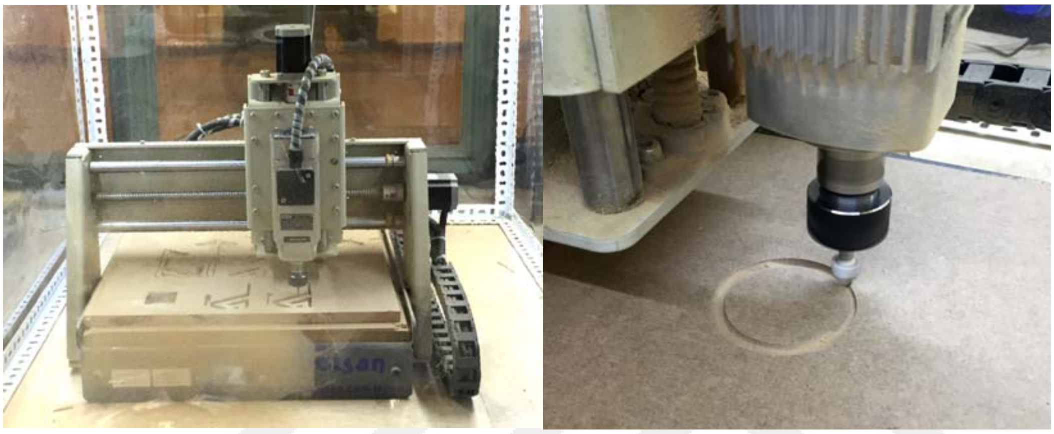

Figure 5.1 : (Left) The 3-axis CNC milling machine used in this study; (Right) Engraving through a circular path with a v-shaped milling tool.

If the parts are defined by points, lines or curves on the digital model, the depth parameter is to be defined by the user. Therefore, different depth values can be used on a single initial geometry that consists of points, lines and/or curves. By defining different depth values to different parts, various parameters can be generated.

If the parts are defined through surfaces on the digital model, the depth value is defined as the coordinates of the surface geometry. Additionally, the direction and of the flow of the tool on the material surface can be defined as a pattern. Some preset carving pattern options include linear, zig-zag and spiral patterns. Many other options can be custom scripted in the G-code. Moreover, the distance between the paths on a toolpath pattern can be defined as the step distance (stepover) of the tool. The stepover is usually defined as very small distances in order to get smooth surfaces without any material left in the carved areas. However, the stepover parameter can be changed according to what kind of a surface the user wants to articulate on the material. For example, stepover values that are larger than the tool diameter will result in various kind of scalloping surface textures depending on the stepover value and the motion pattern of the toolpath.

Lastly, the tool parameters such as shape, diameter, length, cutting length, holder length, and holder diameter need to be defined in order to finalize and generate the toolpath as a G-code. If the parts are defined by points, the tool shape and diameter will determine the shape and the radius of the holes on the surface. If the parts are defined by lines or curves, the tool shape and the tool diameter will determine the shape and the width of the carved volume on the surface. If the parts are defined through surfaces, the tool shape and diameter will determine again the shape of the surface. The tool will carve only the parts that it can reach. For example, if some parts of the surfaces, such as corners and concavities, are smaller than the tool diameter; those parts can not be carved with that particular tool. As a result, the resulting material shape will differ from the initial surface geometry in the digital model.

After the toolpath is generated, the milling processes can be simulated. The simulations enable users to get a basic idea about the resulting shapes prior to the production. In this way, users can experiment with different parameters for form-finding purposes without actually milling each experiment. Today's technology allows the users to obtain simulations that show fine details very quickly. However, the toolpath computations and their simulations are not usually materially informed. Therefore, the results may differentiate at the end of the milling process according to different material behaviors. Various materials such as different kinds of wood, metal, styrofoam, plexiglass etc. can be carved by the same machining operations. Yet, users need to consider material behaviors when choosing some milling parameters such as feed and spindle speed, and all other parameters mentioned before that effect how the material is formed.

In view of all that has been mentioned so far, one can generate and produce various geometric patterns by generating different milling toolpaths from the digital model of a single pattern and these toolpaths are generated by using different milling parameters. By all means, there are many other parameters controlled in the milling processes that have an impact on the formation of the resulting shapes. A full discussion of generative aspects of the making of CNC Milling lies beyond the scope of this study. The case study is focused on the parameters that were analyzed in the making of carved geometric patterns in the previous chapter. These common parameters are depth, distance, tool shape, tool diameter, together with the definition of the parts and the surface geometry.

The study is comprised of two parts. Firstly, experiments for making the two of the existing patterns were conducted with the CNC milling machine in order to examine the generation and application processes using the making rules of Seljuk geometric patterns. The processes were formally represented in the form of making rules and then applied using CNC milling tools. The second part explores a new methodology to generate new patterns by using shape rules and making rules of Seljuk geometric patterns carved into stone by using a single initial shape.

5.1 Experiments on Using Making Rules for Generating and Producing Patterns

The first experiments were focused on the carving parameters, which are tool shape, tool diameter, depth, and distance. The initial shape is chosen as the layout geometry of Pattern I and II from the previous chapters in order to examine the application of different existing patterns from the same initial layout geometry.

The application of Pattern I was divided into three phases (Figure 5.2). Firstly, an engraving operation was applied along the initial lines with a particular distance and depth. Thus, the intermediate level on the existing level was generated. The resulting shape defined the boundaries of the next carving applications are generated. Therefore, this sequence was considered a possible scenario for original generation of this existing pattern. Secondly, inside of the newly generated boundaries were carved with a depth, that is twice the depth of the first phase. Lastly, the highest level of the rest of the parts of the existing pattern has been carved to generate the curved surfaces. All three phases were applied with the same flat-shaped milling tool with a diameter of 2 mm. The roughing of the second phase could have been applied with a wider tool in order to speed up the process, but the finishing of the formation process should be done with the smallest tool possible in order to get the corners of the carved areas as sharp as possible. However, the experiment was conducted on a small scale, so the use of the small milling tool for the roughing part did not cause a major delay.

The material that the pattern is applied on is an MDF (Medium-density fibreboard) is a type of an engineered and homogenized material made out of wood fibers. The stepover value was set at the %30 of the tool diameter in order to get a quick result. The traces of the toolpaths could be seen during the process and the on the end product (Figure 5.3). The texture can be removed with sandpaper in order to get a smooth surface as it is on the example existing pattern. The application can be further examined and developed using ball-shaped or other types of milling tools.

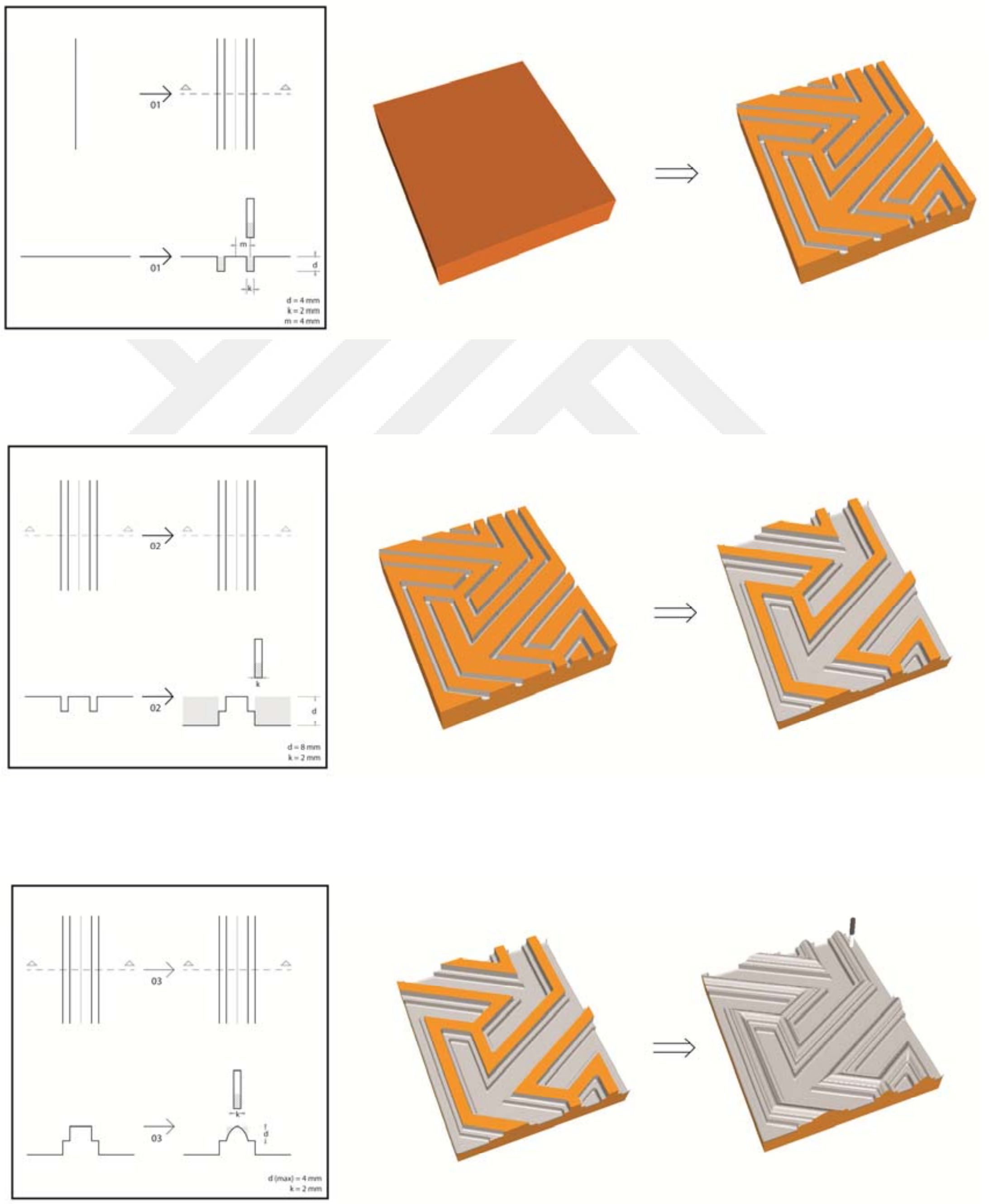

Figure 5.2 : Three sequential making rules of Pattern I and the simulated results of the CNC milling toolpath computation based on the rules.

Figure 5.3 : (First row) Images of the three-phased production of Pattern I; (Second row) Images of the resulting pattern.

The application of Pattern II was divided into two phases (Figure 5.4). Firstly, an engraving operation was done around the lines of the initial pattern. In this case, a V-shaped milling tool was used in order to get the carved geometry on the example pattern. In this case, tool diameter changes according to the carving depth. The milling tool with an angle of 60° was chosen considering these both dimensions. Secondly, the curved areas on the surface were generated by using a flat-shaped milling tool. Images of this application, the V-shaped tool and the resulting pattern can be seen in Figure 5.5.

Similar to the first application, the processes were started with the engraving of lines in order to generate the boundaries to be defined as parts at the next phases. This sequencing can also be meaningful for the hands-on carving applications of these patterns. In this way, the resulting geometries can be generated by material transformations as a paperless process. The carving tools are kind of drawing tools that generate three-dimensional forms manually and also digitally on the simulations. In this way, the perception of the becoming forms can be part of the generative processes of the pattern designs.

The last example is an experiment on pattern generation derived from the tool shape parameter. This particular experiment is not an application of an existing pattern. An engraving operation was applied on the typical circular grid in Seljuk geometric patterns with six-fold rotational symmetry. The use of a V-shaped tool for engraving the intersecting shapes on the pattern resulted in a pattern with various shapes. The images from the application process, the resulting pattern, the making rule and the representation of the overall computation can be seen in Figure 5.6. As a result of the making process, the initial linear geometry has transformed into three different closed shape boundaries on the highest level of the product. The result of this experiment shows that various shapes can emerge as a result of the making computation even with a single basic making rule.

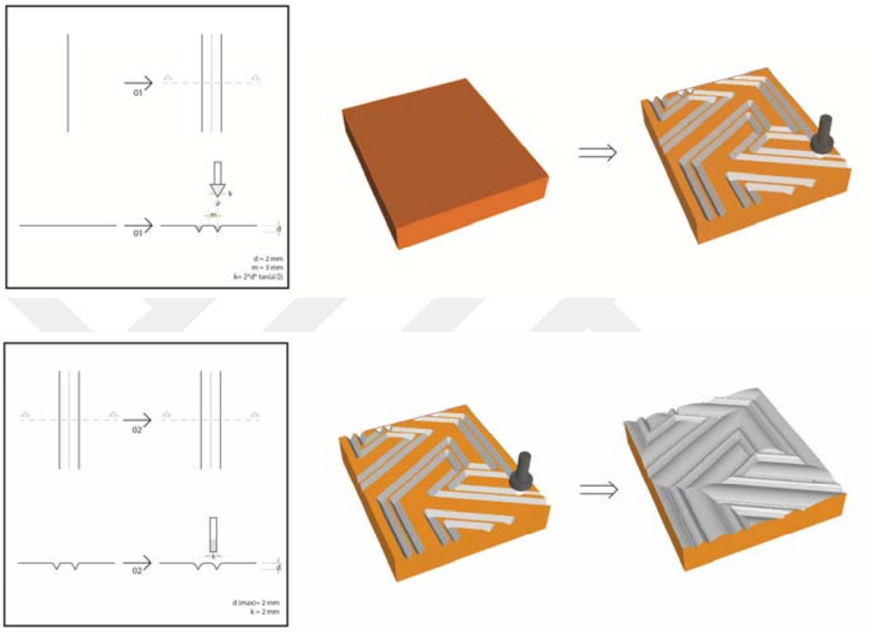

Figure 5.4 : Two sequential making rules of Pattern II and the simulated results of the CNC milling toolpath computation based on the rules.

Figure 5.5 : (First row) Images of the two-phased production of Pattern II; (Second row) Images of the v-shaped milling tool and the resulting pattern.

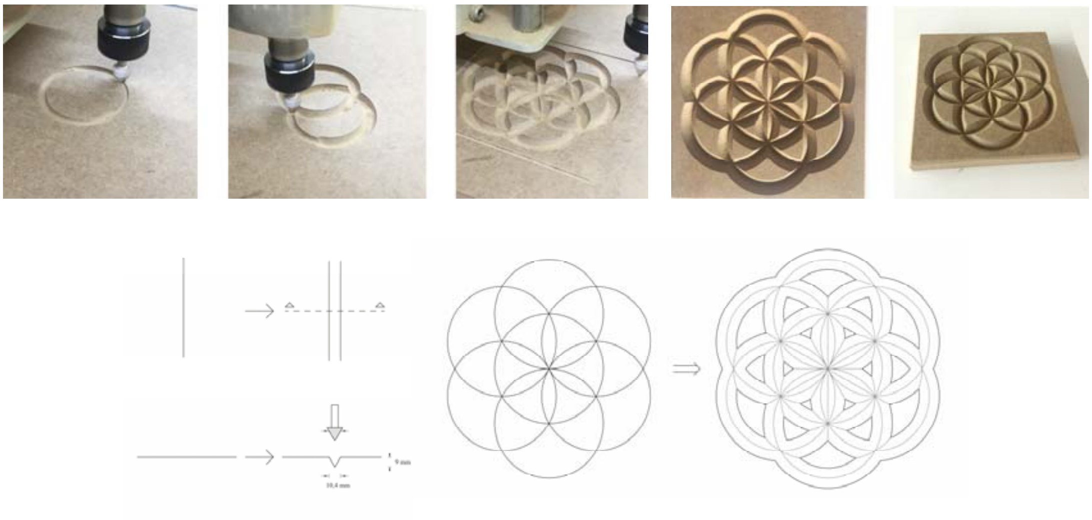

Figure 5.6 : (First row) Images of the one-phased CNC milling production and the resulting pattern; (Second row) The making rule and computation of the process.

5.2 Experiments on Integrating Shape Rules and Making Rules for Generating and Producing Patterns

Experiments conducted in this section aims at integrating the shape rules and making rules in digital fabrication. Two different pattern layouts from the Seljuk-era were used as initial shapes in two different one-day workshops[2]. The participants used different milling tools for carving the wood panels. The simulations provided the participants an insight of the formation of the models and the resulting forms.

The participants of the first workshop were twelve undergraduate and graduate students from different design disciplines such as urban design, industrial design, and architecture. They were provided with the necessary background information on Seljuk geometric patterns and then they were introduced to the basics of generating toolpaths for the CNC milling production. The participants were divided into four groups. All groups were provided with the same initial geometry.

At the first phase, the participants transformed the initial geometry by using shape rules. They were expected to use maximum two shape rules. The shape transformation options were addition and subtraction of some parts on the initial

[2] The first workshop was titled “Seeing and Making Geometry”. More information about the workshop can be obtained on its website: https://makinggeometry.blogspot.com.tr/ . The second workshop was titled “How do we compute design making?” More information about the workshop can be obtained in its website: https://cdmworkshops.wordpress.com/.

shape without changing the rotational symmetry of the pattern. The first two groups used only addition rules, whereas the last two groups used both addition and subtraction rules. As a result, four different patterns were generated from the same initial pattern.

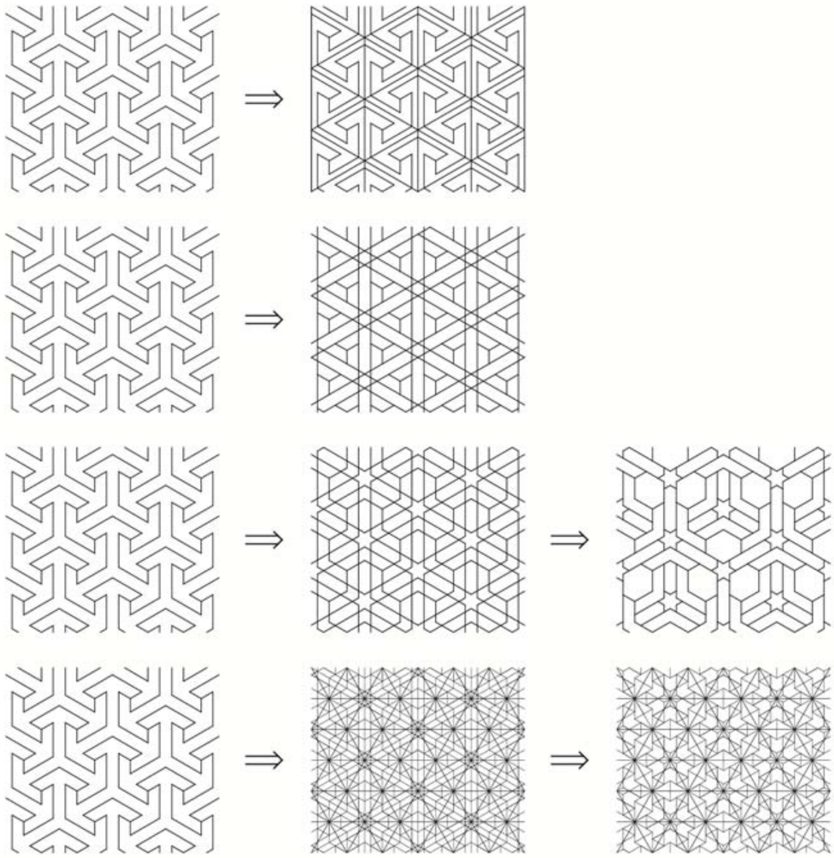

Figure 5.7 : Transformations from the same initial pattern into four different patterns using shape rules.

In the second phase, participants were expected to generate toolpaths by using the making rules to generate and produce the patterns. They were provided with the information of the available milling tools and the base material. The participants then developed patterns by experimenting with different tools, milling operations, and part definitions. The simulations enabled the participants to gain insight about the result of the toolpaths they generated.

After the workshop, the resulting patterns and their generation processes in the were analyzed as a series of making rules. Figure 6.8 shows the analysis of the first two outcomes. Each row describes one phase of each process in terms of part definitions from the initial shapes, making rules, resulting boundaries of the material shape, simulation results and physical models.

The first pattern was generated as a result of two phases. In the first phase, the linear paths are defined and engraved with a flat-shaped milling tool. The tool diameter was equal to 10 mm and the depth was equal to 6 mm. As a result, triangular islands emerged on the material surface. In the second phase, the parts are defined again as lines and engraved with a flat-shaped milling tool. However, in this instance, the tool diameter was smaller and equal to 2 mm, and the carving depth was equal to 3 mm. Consequently, the second operation carved through the triangular island tops on the material surface and generated trapezoid shaped boundaries on the highest level of the surface. In both phases, a single making rule was applied to all lines on the defined parts and as a result, the geometric layout of the pattern is transformed by the making process.

The second pattern was generated as a result of three phases. In the first phase, the toolpath was defined as linear parts. The linear paths were engraved with a flat-shaped milling tool. The carving depth was equal to 2 mm and the tool diameter was equal to 10 mm. The result was similar to the first phase of the first pattern, but in this instance, the depth is smaller. As a result, triangular island tops resulted on the material surface. In the second and third phases, different trapezoid-shaped parts were defined as planes and carved by a flat-shaped milling tool with different depths at each phase. The resulting pattern consisted of four levels and the geometric layout of its initial shape was transformed by the making rules.

The analysis of the third and fourth patterns was presented in Figure 6.9. In these cases, the second columns in each row show the placement of the defined planar parts on particular surface geometries. Since the 3-axis milling machine is able to subtract the reachable parts of the top of these planes, the transformations of the carved material shapes are related to the different surface geometries and tool choices.

The third pattern was generated as a result of three phases. In each phase, the parts were placed on surfaces with particular slopes. Different patterns were generated sequentially at the end of each phase. The computations of these pattern generations are related with many parameters such as surface geometry, tool shape and tool diameter, and the resulting form of the previous phase. Therefore, the pattern generation was hardly predictable for the participants. The simulations helped the participants to gain insights about the emerging material shapes.

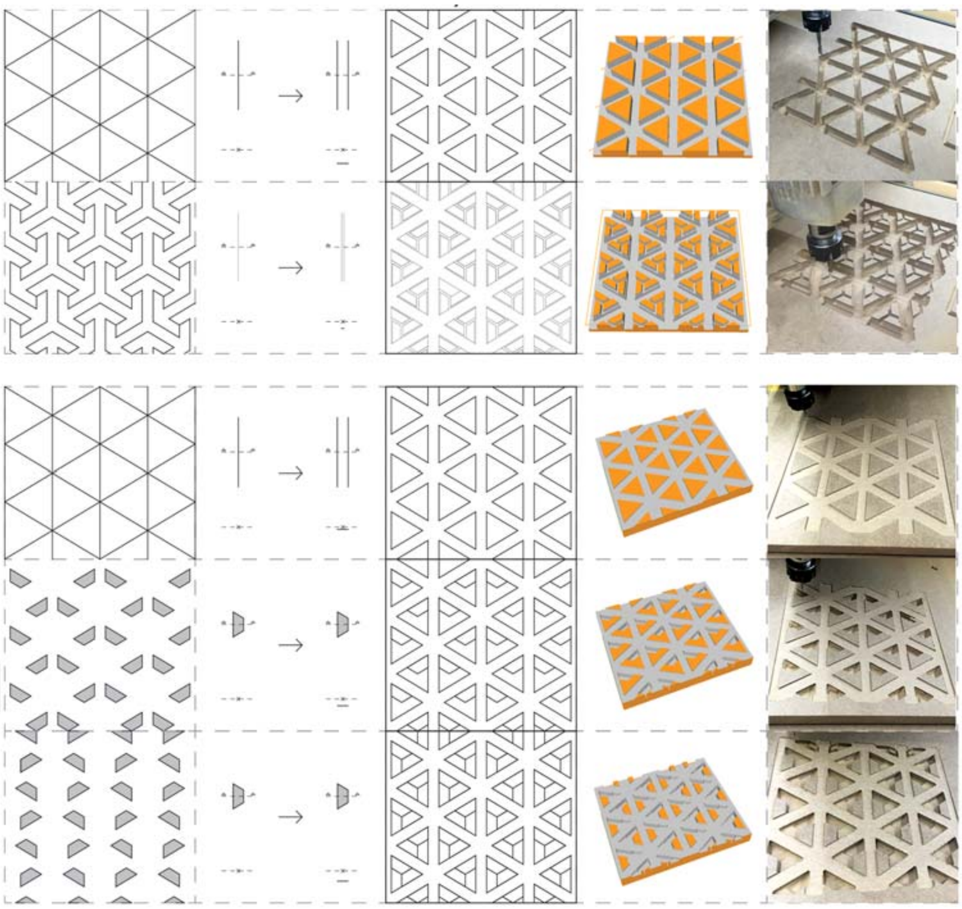

Figure 5.8 : (First two rows) Generation process of the first pattern; (Last three rows) Generation process of the second pattern.

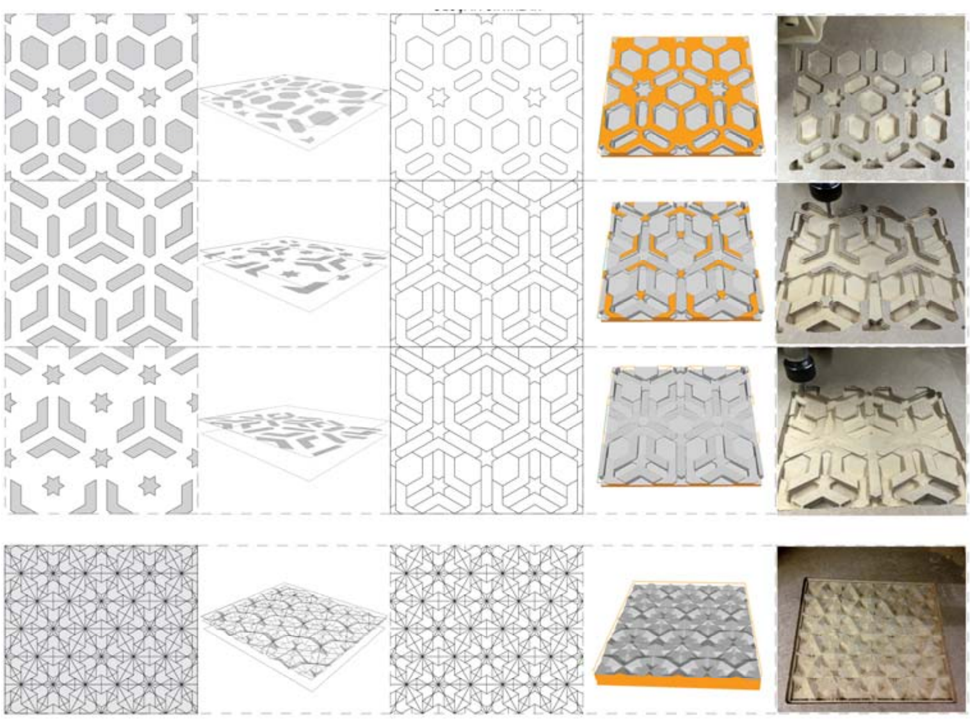

Figure 5.9 : (First three rows) Generation process of the third pattern; (Last row) Generation process of the fourth pattern.

The fourth pattern was generated as a result of a single phase. The planar parts were defined and placed on a digitally modeled surface geometry. In this instance, the surface geometry included sharp angled concavities at the lowest level. The smallest tool diameter that could be used for the 3-axis machine is equal to 2 mm. Therefore, the tool could not reach to the corners of the concave parts. As a result, circle-shaped flat areas with a diameter of 2 mm were generated instead of sharply angled concavities on the surface. This instance highlights the difference between digital and physical generations of a pattern design.

Figure 5.10 shows the making rules of the third and fourth patterns. The first rule indicates the scalloping texture that the participants in the third group generated in one of their experiments. The simulation of the resulting patterns can be seen in the same figure. The pattern on the left was generated by using the tool with a diameter of 2 mm and the pattern on the right was generated by using the tool with a diameter of 5 mm. As a result, the surface textures are different. Moreover, the second making rule indicates the differentiation of the resulting vertical section by carving of sharp angled concavities with a flat-shaped tool.

In the second workshop, a different geometric pattern was used as the initial shape. The participants were introduced to the six transformation rules that are also presented in the third chapter of this thesis study. The aim was to enhance the integration of the use of shape rules and making rules by enabling the participants to gain control of the both processes. For example in the first phase, the participants were expected to transform the patterns by formalizing the generation process at the same time. Figure 5.11 shows the shape rules of one example student work and the shape computation process that were done with these rules.

Similar to the first workshop, participants were asked to generate and produce material shapes based on the final result of the shape transformations done in the first phase. Figure 5.12 shows the generation process of the example pattern in five phases. At each phase, different making rules were applied to different parts defined as lines or planes. The making rules differentiate from each other in terms of tool shape and carving depth. The first three phases were applied with a flat-shaped tool and the last two phases were applied with a V-shaped tool. At the end of the five phases various material shapes emerged at the intersection points of the initial geometry (Figure 6.13).

The methodology, as illustrated in the results of the two workshops on existing patterns, unifies the components of making and design, and expands the design space of any pattern with the factors of production. The results suggest that using making rules enable designers to reason about the complex process of making and its emergent results. For example, the use of various depths and surface geometries diversifies the emergent surface forms by generating various lateral faces on the material shapes. The method can be considered as both a form-making and a form-finding study informed by the generative aspects of the making.

Moreover, it is also worth noting that, integrating shape rules and making rules enhance the generative process of making by expanding the variations in the design space. The main limitation of the method was the one time only use of shape rules and making rules. Yet, the design generation and ideation can be further developed by integrating the feedbacks from the making process by returning back to the shape transformations again in future studies.

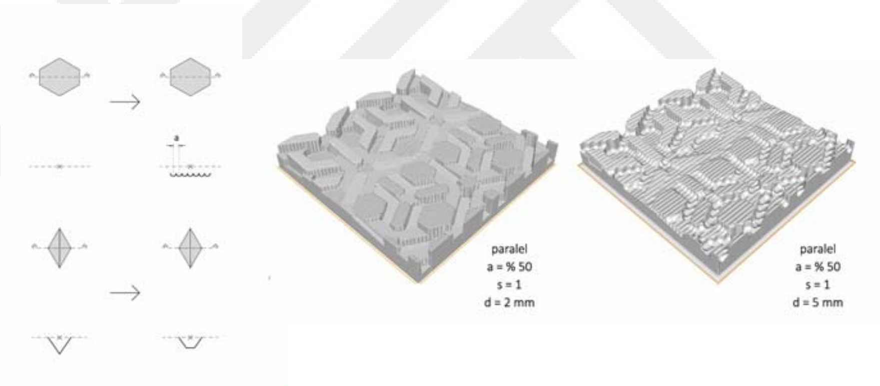

Figure 5.10 : (Left) The making rules of third and fourth patterns (Right) The generation of two different surface textures by using different milling tools.

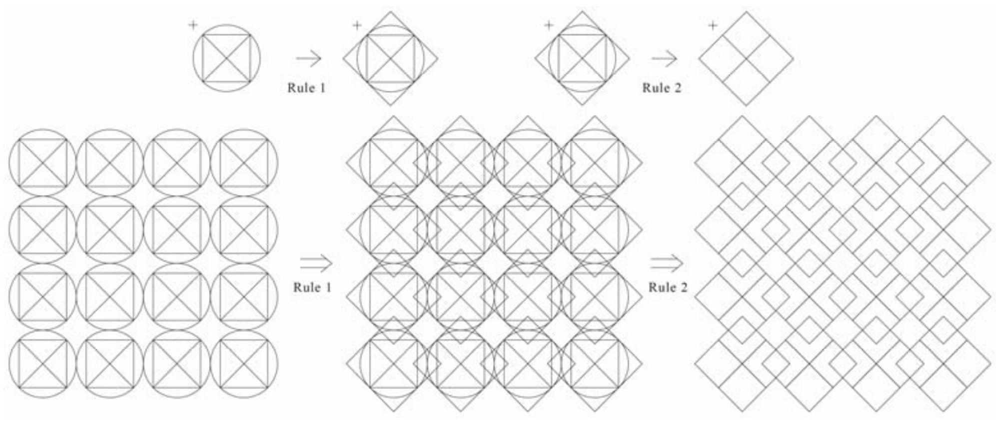

Figure 5.11 : The shape transformations of the example pattern.

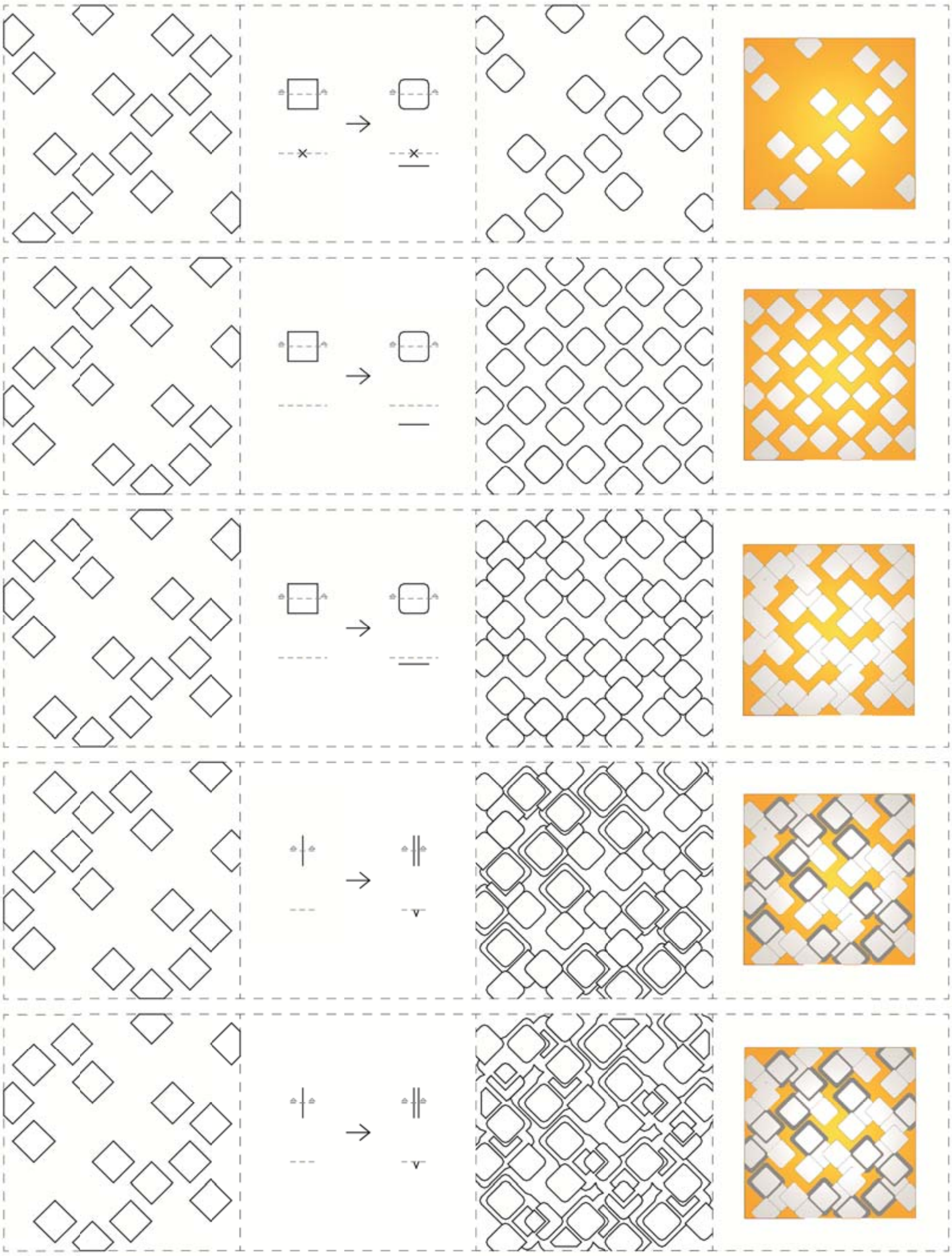

Figure 5.12 : Generation of the pattern as a result of the milling process.

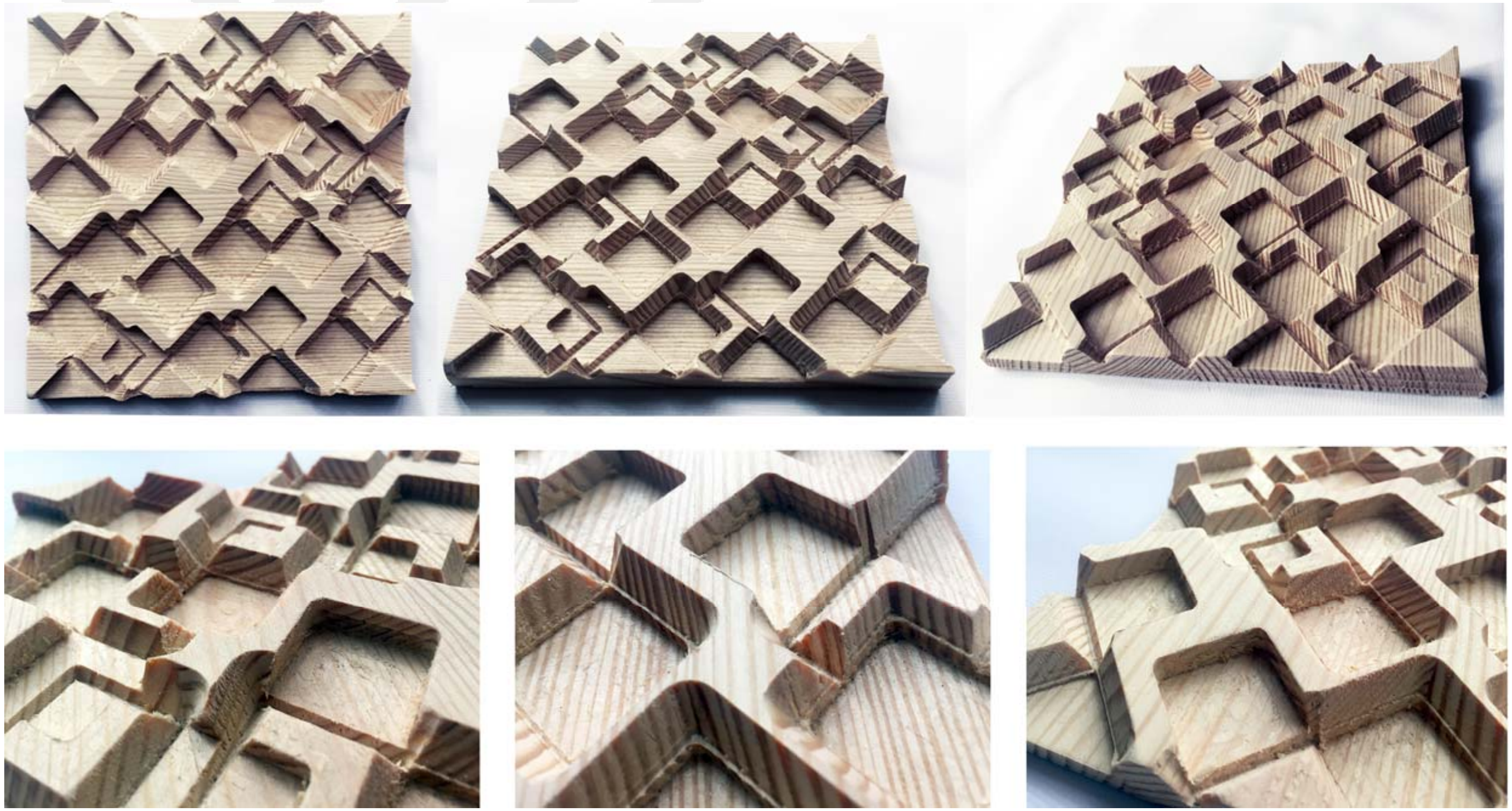

Figure 5.13 : Images of the resulting material shape and the detailed images showing the various carved forms where different parts intersect on the initial shape.

6. CONCLUSION

This study investigated how the design generation and the making of stone carved Seljuk geometric patterns correlate. A selection of patterns from the Seljuk-era has been analyzed in terms of transformations that generate the final patterns. The significance of this study lies in the analysis of patterns as material things rather than pure geometries. The issue of analyzing the progressive generation process of the carved patterns on finished artifacts has been addressed. Firstly, a method for reasoning in the layout generation of patterns has been developed based on the integration of the knowledge of making and visual computation. The underlying geometric compositions, i.e. the initial shapes of the making of the patterns have been examined by illustrating the generation processes as a series of shape computations starting from the circle grids. The distinction of the method lies in the consideration of possible application and transformation scenarios that may generate the final patterns. The method uses circular grids that match with the symmetry group of the patterns as the base point and therefore reveals the relationship between the visible boundaries on the stone carved patterns and the possible initial structure. These relations were then examined later as material transformations of the making process. Secondly, the algebraic transformations of the compass-straightedge construction of the geometric shapes from circular grids have been represented in the form of shape rules. Seven different types of general transformation rules (tessellation, addition, subtraction, translation, extrusion, rotation, curving) have been highlighted. The results revealed that the ambiguity in the part relations of the shapes and various algebraic transformations are related to the emergence of these rich geometric compositions. Thus, this research presented a circular grid-based method for analyzing the Seljuk geometric patterns and by doing so, allowed the integration of many other possible variations in the design ideation of these patterns. This approach has the potential to contribute to the development of a comprehensive shape grammar for Seljuk geometric patterns based on visual schemas. In that case, a further study comprising many example patterns in other monuments from the

Seljuk-era would be needed. This study can also be expanded by comprising the patterns made with different materials such as brick, ceramic or wood.

Moreover, the analysis of the shape generation process revealed that patterns can share the same design as the initial shape even when they appear differently. Therefore, it has been showed that there are specific material transformations involved in the design ideation of Seljuk geometric patterns. These findings led to the idea of developing a making grammar approach to externalizing these transformations and understanding the design generation process of the stonemasons from the Seljuk-era. A rule-based computational making method for illustrating possible scenarios of these applications were presented.

The formalization study of the making rules has led to the following conclusions. Firstly, tool-based design generation and emergence have been introduced as new concepts that emerged from the rule-based computational making method presented in this study. The research on tool-based emergence can be further developed by examining design generations emerged from various tool parameters with a computational approach. This approach may bring new meanings to the concept of digital craft as well. Secondly, abstracting the boundaries of the carved patterns as horizontal and vertical sections can be useful for tracing the transformations of carved geometries. The lines in the section drawings work as shapes in visual rules and therefore represent the ambiguous and emergent nature of the material transformations. Yet certainly, this formalization cannot be used for different materials with different behaviors. Thirdly, the formalizations of geometrical constructions on curved surfaces using different tools show that tools need to be represented with specific formal algebras. The formal algebras of tools depend on the particular action in which they are used as well. For example, wrapping paper on a curved surface can be described with U_{23} algebra, whereas bending a folded paper can be described with U_{33} algebra and on the other hand wrapping a string on a curved surface can be described with U_{13} algebra. Hence, algebraic formalizations of different tools can be useful for specifying and categorizing making computations and therefore reasoning about the relations such as the one between the surface geometry and the making rules. The matrix of practical relations in the making of geometric patterns can be further investigated and integrated into the making computation. Furthermore, one of the main differences between computing with material things and shapes is that making computations may not be reversible. In other words, materials do not fuse as shapes do. For instance, the transformations in the case of stone carving are not reversible. Consequently, the order of the actions is important. In this study, the cut order was indicated with numbers within the rules. On the other hand, timestamp was not relevant in the computations analyzed in this study since the actions do not happen simultaneously and stone is a stabilized matter. Yet, the cut order signifies a time-based grammar as well.

The applications at the end of the study implicate that, making rules can be useful for enhancing computational design processes and the use of digital fabrication tools limited to automating form generation by means of diversity and integrity. The significant contribution of this model lies in the integration of the knowledge of making to the design ideation. In the case study, the main focus has been on the cutting tool parameters such as tool diameter and tool shape, and the milling parameters such as depth. The results suggest that integrating tool parameters of digital fabrication tools in design ideation process can lead to tool-based emergence. This integrity may bring new meanings to the CNC milling and other digital fabrication aided design processes. Furthermore, the experiments of using shape rules and making rules together showed that these two generative processes can enhance each other in a mutual way. The outcomes of the experiments conducted in two student workshops revealed that it would be useful to apply shape rules and making rules in a more cyclical order. Then it would be possible to enhance the geometrical compositions as initial shapes based on the results of the material transformations. The reason is that making is such a complex process and the results of its computations often cannot be predicted until the end of the experiments. The simulations, such as the ones for simulating CNC milling codes, can assist designers by enabling faster decision making. However, the simulations do not reflect all material behaviors and moreover, the digital fabrication tools can bring unexpected results since the digital fabrication machines are not yet capable of recognizing failures such as misinformation and misplacement of the material. On the other hand, combining shape rules with making rules in computational design, essentially concurs well with Knight and Stiny's (2015) argument that "design is a kind of making". In that context, the whole process can be regarded as making.

The rule-based computational making presented in this study may bring new meanings to the stone carved geometries from the Seljuk era by introducing the impacts of the parameters such as tool shape, tool diameter, carving depth, carving distance, boundary definition and surface geometries. This approach could be useful for developing a more comprehensive making grammar for stone carved geometric patterns. The literature can be expanded in numerous ways. For example, it would be useful to investigate the making knowledge of stone carved patterns in the geographically and chronologically adjacent cultures. Furthermore, one of the most significant roles of the rule-based approach presented in this study is to reveal formal relations between pattern layout geometry, surface geometry, rotational symmetry, initial shape, location and its place in the cultural history. These findings enhance our understanding of the design generation of stone carved Seljuk geometric patterns. The study also suggests a possible contribution to the restoration applications of the existing Seljuk geometric patterns. Integrating the knowledge of the making of the patterns may prevent improper restoration applications based only on the analysis of the final forms of the existing patterns. Although the current study is based on a small sample of patterns, the open access database of this research is to be expanded to serve as a base for future collaborations with historians and restorators. Therefore, this thesis represents the initial stage of a more comprehensive study of the cultural heritage of the craft based on the rule-based computational making approach. Future studies should target different carving tools and methods including the ones for making particular textures with various kinds of stones. Furthermore, the research on the algebraic formalizations of the materials, tools and the actions and their relations in the computational making of geometric patterns will be the next challenge.

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CURRICULUM VITAE

Name Surname : **Begüm HAMZAOĞLU**

Place and Date of Birth : **İstanbul, 30.10.1990**

E-Mail : **hamzaoglu@itu.edu.tr**

EDUCATION :

- **B.Arch.** **:** 2014, Istanbul Technical University, Faculty of Architecture

PROFESSIONAL EXPERIENCE AND REWARDS:

- Istanbul Technical University, Faculty of Architecture (May 2017 – present) Position: Research Assistant

PUBLICATIONS/ PRESENTATIONS ON THE THESIS:

- **HAMZAOĞLU, B., ÖZKAR, M. (**2016). Tasarım ve Yapmanın İlişkisi: Selçuklu Geometrik Desenleri Örneğine Güncel Bir Bakış. MSTAS 2016 Karşılaşmalar: Krizler ve İmkanlar [10. Mimarlıkta Sayısal Tasarım Ulusal Sempozyumu], İstanbul Bilgi Üniversitesi, İstanbul, 58-69.

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