

ENGINEERING MATHEMATICS MIDTERM I SOLUTIONS

1) $(t^2 + y^2)dt + 2ty dy = 0$ $y(t_0) = y_0$
 Here $M = t^2 + y^2$ $N = 2ty$ Homogeneous of degree 2
 $y = ut$ $dy = t du + u dt$
 $(t^2 + u^2 t^2)dt + 2t^2 u(t du + u dt) = 0$, $(1 + 3u^2)dt + 2t u du = 0$, $\int_{u_0}^u + \int_{t_0}^t = 0$
 $\frac{1}{3} \left[\ln \frac{1+3u^2}{1+3u_0^2} \right] + \ln \frac{t}{t_0} = 0$ $u = \left[\frac{1}{3} \frac{[(1+3u_0^2)(\frac{t_0}{t})^3 - t^3]}{t^3} \right]^{1/2}$
 $y = \left(\frac{(1+3y_0^2)t_0^3 - t^3}{3t} \right)^{1/2} = \left(\frac{t_0^3 + 3y_0^2 t_0 - t^3}{3t} \right)^{1/2} = \sqrt{\frac{t_0(t_0^2 + 3y_0^2) - t^3}{3t}}$

2) Differentiate
 $2(s-x) \left(\frac{ds}{dt} - \frac{dx}{dt} \right) + 2y \frac{dy}{dx} \frac{dx}{dt} = 0$
 $(200t-x) \left(200 - \frac{dx}{dt} \right) + y \left(-\frac{y}{s-x} \right) \frac{dx}{dt} = 0$ $(s-x) \left(200 - \frac{dx}{dt} \right) - \frac{y^2}{(s-x)} \frac{dx}{dt} = 0$
 $-\frac{y^2}{s-x} \frac{dx}{dt} = \frac{(s-x)^2 - 25}{(s-x)} \frac{dx}{dt}$ Eliminate $\frac{dx}{dt}$
 $\frac{dx}{dt} = 8(s-x)^2 = 8(200t-x)^2$
 $\frac{dx}{dt} = 8u^2$
 $u = 200t - x$
 $x = 200t - u$

$\frac{dx}{dt} = 200 - \frac{du}{dt}$
 $\frac{du}{dt} = 200 - \frac{dx}{dt} = 200 - 8u^2 = 8(5-u^2)$
 $\frac{du}{25-u^2} = 8dt$
 $\frac{1}{10} [\ln(5+u) - \ln(5-u)] = 8t + C$ $t=0$ $u=0$ $C=0$
 $x = 200t - u = 200t - 5 + \tanh 40t$
 $\frac{dy}{dt} = -\frac{y}{u} \frac{dx}{dt} = -8uy$, $\frac{dy}{y} = -8u dt = -40 \tanh 40t dt$
 $\ln y = \ln \cosh 40t + k$ $t=0$ $y=5$
 $y = 5 \operatorname{sech} 40t$

3) $(A - \lambda I)X = 0$
 $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} a-\lambda & b \\ -b & a-\lambda \end{bmatrix}$ det $\rightarrow (a-\lambda)^2 + b^2 = 0$, $\lambda^2 - 2a\lambda + a^2 + b^2 = 0$
 $\lambda_1 = a+ib$ $\lambda_2 = a-ib$
 for λ_1 $\begin{bmatrix} a-(a+ib) & b \\ -b & a-(a+ib) \end{bmatrix} \sim \begin{bmatrix} -ib & b \\ 0 & 0 \end{bmatrix}$ $x_1 = 1$ $x_2 = i$ $\begin{pmatrix} 1 \\ i \end{pmatrix}$
 for λ_2 $\begin{bmatrix} a-(a-ib) & b \\ -b & a-(a-ib) \end{bmatrix} \sim \begin{bmatrix} ib & b \\ 0 & 0 \end{bmatrix}$ $x_1 = 1$ $x_2 = -i$ $\begin{pmatrix} 1 \\ -i \end{pmatrix}$

3)

$$\begin{array}{c|cccc|c}
 m_{i1} & 1 & 2 & 3 & 4 & 6 \\
 1 & 1 & 4 & 9 & 16 & 22 \\
 1 & 1 & 8 & 27 & 64 & 84 \\
 1 & 1 & 16 & 81 & 256 & 322
 \end{array}
 \rightarrow
 \begin{array}{c|cccc|c}
 m_{i2} & 1 & 2 & 3 & 4 & 6 \\
 0 & 0 & 2 & 6 & 12 & 16 \\
 3 & 0 & 6 & 24 & 60 & 78 \\
 7 & 0 & 14 & 78 & 252 & 316
 \end{array}
 \rightarrow$$

$$\begin{array}{c|cccc|c}
 m_{i3} & 1 & 2 & 3 & 4 & 6 \\
 0 & 0 & 2 & 6 & 12 & 16 \\
 0 & 0 & 6 & 24 & 30 & 30 \\
 6 & 0 & 0 & 36 & 168 & 204
 \end{array}
 \rightarrow
 \begin{array}{c|cccc|c}
 1 & 1 & 2 & 3 & 4 & 6 \\
 0 & 0 & 2 & 6 & 12 & 16 \\
 0 & 0 & 6 & 24 & 30 & 30 \\
 0 & 0 & 0 & 24 & 24 & 24
 \end{array}
 \rightarrow
 \begin{array}{l}
 x_1 = 1 \\
 x_2 = -1 \\
 x_3 = 1 \\
 x_4 = 1
 \end{array}$$