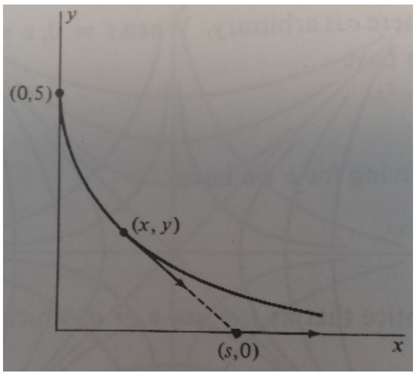
30 October 2018

Engineering Mathematics

1) Solve the equation below

$(t^2 + y^2)dt + 2tydy = 0 \qquad y(t_0) = y_0$ assuming $t_0$ and $y_0$ positive.

2) An airplane is flying in a straight line with a constant speed of 200 miles per hour. A second plane is initially flying directly toward the first on a line perpendicular to its path. The second plane continues to pursue the first in such a way that the distance between the planes remains constant (5 miles) and the pursuing plane is always headed toward the other; that is, the tangent to the path of the pursuer passes through the other. Consider the problem in the *x-y* plane.




Let the coordinates of the pursuing plane be (*x,y*) and the coordinates of the other be (*s,0*). The conditions of the problem can be stated in the following equations

$$(s-x)^2 + y^2 = 25$$

$$\frac{dy}{dx} = -\frac{y}{s-x}$$

$$s = 200t$$

Find *x* and *y* as functions of *t* subject to the initial conditions at *t*=0; *s*=0, *x*=0, *y*=5.

3) Find the general solution of the system of linear algebraic equations given below.

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 6$$
$$x_1 + 4x_2 + 9x_3 + 16x_4 = 22$$
$$x_1 + 8x_2 + 27x_3 + 64x_4 = 84$$
$$x_1 + 16x_2 + 81x_3 + 256x_4 = 322$$

4) Find the linear independent eigenvectors of the following matrix.

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

(*a, b* real, *b*≠ 0)

Reminder: $\int \frac{du}{u^2-a^2} = \frac{1}{2a} ln \frac{u-a}{u+a} + c \quad u^2 > a^2$

$\int \frac{du}{u^2-a^2} = \frac{1}{2a} ln \frac{a-u}{a+u} + c \quad u^2 < a^2$

$tanhx = \frac{e^{2x}-1}{e^{2x}+1}$