The extension Chebyshev polynomial bounds for general subclasses of bi-univalent functions involving subordination

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Abstract

In this present investigation, we use the trigonometric polynomials $U_n(q, e^{i\theta})$ to get the initial coefficients of bi-univalent functions in the new-defined classes $\mathfrak{S}_{\Sigma}^{a,b,c}(\xi, q, \theta)$ and $\mathfrak{K}_{\Sigma}^{a,b,c}(\xi, q, \theta)$. We also derive Fekete-Szegö inequalities for functions in these classes.

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1 Introduction

Let $\mathbb{R} = (-\infty, \infty)$ be the set of real numbers, $\mathbb{C}$ be the set of complex numbers and

$$\mathbb{N} := \{1, 2, 3, \ldots\} = \mathbb{N}_0 \backslash \{0\}$$

be the set of positive integers. Let $D = \{z \in \mathbb{C} : |z| < 1\}$ be open unit disc in $\mathbb{C}$. As is well known, the trigonometric polynomials $U_n(q, e^{i\theta})$ are expressed by the generating function

$$\begin{aligned}\Psi_q(e^{i\theta}, z) &= \frac{1}{(1 - ze^{i\theta})(1 - qze^{-i\theta})} \\ &= \sum_{n=0}^{\infty} U_n(q, e^{i\theta}) z^n \quad (q \in (-1, 1], \theta \in [-\pi, \pi], z \in D),\end{aligned}$$

where

$$U_n(q, e^{i\theta}) = \frac{e^{i(n+1)\theta} - q^{n+1} e^{-i(n+1)\theta}}{e^{i\theta} - qe^{-i\theta}} \quad (n \geq 2)$$

with

$$U_0(q, e^{i\theta}) = 1,\ U_1(q, e^{i\theta}) = e^{i\theta} + qe^{-i\theta},\ U_2(q, e^{i\theta}) = e^{2i\theta} + q^2 e^{-2i\theta} + q, \ldots.$$

The obtained results for $q = 1$ give the corresponding ones for Chebyshev polynomials of the second kind. The classical Chebyshev polynomials which are used in this paper, have been given in the late eighteenth century, when was defined using de Moivre's formula by Chebyshev [7]. Such polynomials as (for example) the Fibonacci polynomials, the Lucas polynomials, the Pell polynomials and the families of orthogonal polynomials and other special polynomials as well as their generalizations are potentially important in the fields of probability, statistics, mechanics, and number theory.

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Let A represent the class of functions f of the form

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots, \tag{1.1}$$

which are analytic in D and normalized under the condition $f(0) = f'(0) - 1 = 0$. Further, we indicate by S the subclass of A consisting of functions that are univalent in D.

With a view to remanding the rule of subordination between analytic functions, let the functions f, g be analytic in D. A function f is subordinate to g, indicated as $f \prec g$ (or $f(z) \prec g(z)$) $(z \in D)$, if there exists a Schwarz function $\mathfrak{w} \in \Lambda$, where

$$\Lambda = \{\mathfrak{w} : \mathfrak{w}(0) = 0, \ |\mathfrak{w}(z)| < 1, \ z \in D\},$$

such that

$$f(z) = g(\mathfrak{w}(z)) \qquad (z \in D).$$

According to the *Koebe-One Quarter Theorem* [8], it provides that the image of D under every univalent function $f \in A$ contains a disc of radius $1/4$. Thus every univalent function $f \in A$ has an inverse f^{-1} satisfying $f^{-1}(f(z)) = z$ and $f\left(f^{-1}(w)\right) = w \ \left(|w| < r_0(f), \ r_0(f) \geq \frac{1}{4}\right)$, where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots. \tag{1.2}$$

A function $f \in A$ is said to be bi-univalent in D if both f and f^{-1} are univalent in D. Let Σ represent the class of bi-univalent functions in D given by (1.1). For a brief historical account in the class Σ, see [19] (see also [1, 2, 3, 4, 5, 6, 9, 13, 17, 18, 20, 21]).

The convolution or Hadamard product of two functions $f, g \in A$ is denoted by $f * g$, and is defined by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n,$$

where f is given by (1.1) and $g(z) = z + \sum\limits_{n=2}^{\infty} b_n z^n$. Next, in our present investigation, we need to recall the convolution operator $\mathfrak{I}_{a,b,c}$ due to Hohlov [14, 15], which is a special case of the Dziok-Srivastava operator [10, 11].

For the complex parameters a, b and c $(c \neq 0, -1, -2, \ldots)$, the Gaussian hypergeometric function ${}_2F_1(a, b, c; z)$ is defined as

$$ {}_2F_1(a, b, c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} = 1 + \sum_{n=2}^{\infty} \frac{(a)_{n-1} (b)_{n-1}}{(c)_{n-1}} \frac{z^{n-1}}{(n-1)!} \qquad (z \in D),$$

where $(a)_n$ is the Pochhammer symbol (or the shifted factorial) given by

$$(a)_n := \frac{\Gamma(a+k)}{\Gamma(a)} = \begin{cases} 1 & n = 0 \\ a(a+1)(a+2)\ldots(a+n-1) & n \in \mathbb{N} := \{1, 2, \ldots\} \end{cases}.$$

Now we consider a linear operator introduced by Murugusundaramoorthy and Bulboaca [16] and

$$\mathfrak{I}_{a,b,c} : A \to A,$$

defined by the Hadamard product

$$\mathfrak{I}_{a,b,c}f(z) = (z_2F_1(a,b,c;z)) * f(z).$$

We observe that, for a function f of the form (1.1), we have

$$\mathfrak{I}_{a,b,c}f(z) = z + \sum_{n=2}^{\infty} \Xi_n a_n z^n \qquad (z \in D),$$

where $\Xi_n = \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}$.

The classical Fekete-Szegö inequality, expressed by means of Loewner's method, for the coefficients of $f \in S$ is

$$\left|a_3 - ta_2^2\right| \leq 1 + 2\exp(-2t/(1-t)) \quad \text{for } t \in [0,1).$$

As $t \to 1^-$, we have the elementary inequality $\left|a_3 - a_2^2\right| \leq 1$. Moreover, the aim of maximizing the absolute value of the functional

$$\Pi_t(f) = a_3 - ta_2^2$$

is called the Fekete-Szegö problem (see [12]).

We want to assert evidently that by using the trigonometric polynomials $U_n(q, e^{i\theta})$, we establish some new subclasses of bi-univalent functions based on subordination. Afterwards, we derive coefficient bounds and obtain Fekete-Szegö inequalities for these classes.

Definition 1.1. A function $f \in \Sigma$ is said to be in the class

$$\mathfrak{S}_{\Sigma}^{a,b,c}(\xi, q, \theta) \quad (0 \leq \xi \leq 1,\ q \in (-1,1],\ \theta \in [-\pi, \pi];\ z, w \in D)$$

if the following conditions are satisfied:

$$\frac{z\left(\mathfrak{I}_{a,b,c}f(z)\right)'}{(1-\xi)z + \xi\mathfrak{I}_{a,b,c}f(z)} \prec \Psi_q(e^{i\theta}, z) \tag{1.3}$$

and

$$\frac{w\left(\mathfrak{I}_{a,b,c}g(w)\right)'}{(1-\xi)w + \xi\mathfrak{I}_{a,b,c}g(w)} \prec \Psi_q(e^{i\theta}, w), \tag{1.4}$$

where the function $g = f^{-1}$.

Definition 1.2. A function $f \in \Sigma$ is said to be in the class

$$\mathfrak{K}_{\Sigma}^{a,b,c}(\xi, q, \theta) \quad (0 \leq \xi \leq 1,\ q \in (-1,1],\ \theta \in [-\pi, \pi];\ z, w \in D)$$

if the following conditions are satisfied:

$$\frac{z\left(\mathfrak{I}_{a,b,c}f(z)\right)' + z^2\left(\mathfrak{I}_{a,b,c}f(z)\right)''}{(1-\xi)z + \xi z\left(\mathfrak{I}_{a,b,c}f(z)\right)'} \prec \Psi_q(e^{i\theta}, z) \tag{1.5}$$

and

$$\frac{w\left(\mathfrak{I}_{a,b,c}g(w)\right)' + w^2\left(\mathfrak{I}_{a,b,c}g(w)\right)''}{(1-\xi)w + \xi w\left(\mathfrak{I}_{a,b,c}g(w)\right)'} \prec \Psi_q(e^{i\theta}, w), \tag{1.6}$$

where the function $g = f^{-1}$.

2 The extension Chebyshev polynomial bounds for the function classes $\mathfrak{S}_{\Sigma}^{a,b,c}(\xi, q, \theta)$ and $\mathfrak{K}_{\Sigma}^{a,b,c}(\xi, q, \theta)$

In this part, we offer to get the extension Chebyshev polynomial bounds for functions in the classes $\mathfrak{S}_{\Sigma}^{a,b,c}(\xi, q, \theta)$ and $\mathfrak{K}_{\Sigma}^{a,b,c}(\xi, q, \theta)$.

Theorem 2.1. Let the function f given by (1.1) be in the class $\mathfrak{S}_{\Sigma}^{a,b,c}(\xi, q, \theta)$.Then

$$|a_2| \leq \frac{\left|e^{i\theta} + qe^{-i\theta}\right| \sqrt{\left|e^{i\theta} + qe^{-i\theta}\right|}}{\sqrt{\left|(\xi-2)\Xi_2^2\left(2e^{2i\theta} + 2q^2e^{-2i\theta} + (\xi+2)q\right) + (3-\xi)\Xi_3(e^{2i\theta} + q^2e^{-2i\theta} + 2q)\right|}},$$

$$|a_3| \leq \frac{\left|e^{i\theta} + qe^{-i\theta}\right|}{(3-\xi)\Xi_3} + \frac{\left|e^{2i\theta} + q^2e^{-2i\theta} + 2q\right|}{(\xi-2)^2\Xi_2^2}$$

for any real number ρ,

$$\left|a_3 - \rho a_2^2\right| \leq$$

$$\begin{cases} \dfrac{\left|e^{i\theta} + qe^{-i\theta}\right|}{(3-\xi)\Xi_3}, & |\rho - 1| \leq X \\ \\ \dfrac{|1-\rho| \left|e^{2i\theta} + q^2e^{-2i\theta} + 2q\right| \left|e^{i\theta} + qe^{-i\theta}\right|}{\left|(\xi-2)\Xi_2^2\left(2e^{2i\theta} + 2q^2e^{-2i\theta} + (\xi+2)q\right) + (3-\xi)\Xi_3(e^{2i\theta} + q^2e^{-2i\theta} + 2q)\right|}, & |\rho - 1| \geq X \end{cases}$$

where $X = \frac{\left|(\xi-2)\Xi_2^2\left(2e^{2i\theta} + 2q^2e^{-2i\theta} + (\xi+2)q\right) + (3-\xi)\Xi_3(e^{2i\theta} + q^2e^{-2i\theta} + 2q)\right|}{(3-\xi)\Xi_3\left|e^{2i\theta} + q^2e^{-2i\theta} + 2q\right|}$.

Proof. Let $f \in \mathfrak{S}_{\Sigma}^{a,b,c}(\xi, q, \theta)$ be given by the Taylor-Maclaurin expansion (1.1). Then, by the definition of subordination, for two analytic functions ψ, φ such that

$$\psi(0) = 0, \ |\psi(z)| = \left|m_1z + m_2z^2 + m_3z^3 + \cdots\right| < 1 \quad (z \in D),$$

$$\varphi(0) = 0, \ |\varphi(w)| = \left|r_1w + r_2w^2 + r_3w^3 + \cdots\right| < 1 \quad (w \in D),$$

we can write

$$\frac{z\left(\mathfrak{I}_{a,b,c}f(z)\right)'}{(1-\xi)z + \xi\mathfrak{I}_{a,b,c}f(z)} = 1 + U_1(q, e^{i\theta})\psi(z) + U_2(q, e^{i\theta})\psi^2(z) + \cdots$$

and

$$\frac{w\left(\mathfrak{I}_{a,b,c}g(w)\right)'}{(1-\xi)w + \xi\mathfrak{I}_{a,b,c}g(w)} = 1 + U_1(q, e^{i\theta})\varphi(w) + U_2(q, e^{i\theta})\varphi^2(w) + \cdots$$

or, equivalently,

$$\frac{z\left(\mathfrak{I}_{a,b,c}f(z)\right)'}{(1-\xi)z + \xi\mathfrak{I}_{a,b,c}f(z)} = 1 + U_1(q, e^{i\theta})m_1z + \left[U_1(q, e^{i\theta})m_2 + U_2(q, e^{i\theta})m_1^2\right]z^2 + \cdots \tag{2.1}$$

and

$$\frac{w\left(\mathfrak{I}_{a,b,c}g(w)\right)'}{(1-\xi)w + \xi\mathfrak{I}_{a,b,c}g(w)} = 1 + U_1(q, e^{i\theta})r_1w + \left[U_1(q, e^{i\theta})r_2 + U_2(q, e^{i\theta})r_1^2\right]w^2 + \cdots . \tag{2.2}$$

Additionally, it is well known that

$$|m_k| \leq 1 \text{ and } \quad |r_k| \leq 1 \quad (\forall k \in \mathbb{N}). \tag{2.3}$$

Now, upon comparing the corresponding coefficients in (2.1) and (2.2), we get

$$(2-\xi)\Xi_2 a_2 = U_1(q, e^{i\theta}) m_1, \tag{2.4}$$

$$(\xi^2 - 2\xi)\Xi_2^2 a_2^2 + (3-\xi)\Xi_3 a_3 = U_1(q, e^{i\theta}) m_2 + U_2(q, e^{i\theta}) m_1^2 \tag{2.5}$$

and

$$-(2-\xi)\Xi_2 a_2 = U_1(q, e^{i\theta}) r_1, \tag{2.6}$$

$$(\xi^2 - 2\xi)\Xi_2^2 a_2^2 + (3-\xi)\Xi_3 \left(2a_2^2 - a_3\right) = U_1(q, e^{i\theta}) r_2 + U_2(q, e^{i\theta}) r_1^2. \tag{2.7}$$

From the equations (2.4) and (2.6), one can easily find that

$$m_1 = -r_1, \tag{2.8}$$

$$2(2-\xi)^2 \Xi_2^2 a_2^2 = U_1^2(q, e^{i\theta})(m_1^2 + r_1^2). \tag{2.9}$$

If we add (2.5) to (2.7), we obtain

$$2\left[(\xi^2 - 2\xi)\Xi_2^2 + (3-\xi)\Xi_3\right] a_2^2 = U_1(q, e^{i\theta})\left(m_2 + r_2\right) + U_2(q, e^{i\theta})\left(m_1^2 + r_1^2\right). \tag{2.10}$$

By making use of (2.9) in (2.10), we get

$$2\left[(\xi^2 - 2\xi)\Xi_2^2 + (3-\xi)\Xi_3 - \frac{(2-\xi)^2 \Xi_2^2 U_2(q, e^{i\theta})}{U_1^2(q, e^{i\theta})}\right] a_2^2 = U_1(q, e^{i\theta})\left(m_2 + r_2\right),$$

which yields

$$a_2^2 = \frac{U_1^3(q, e^{i\theta})\left(m_2 + r_2\right)}{2\left\{\left[(\xi^2 - 2\xi)\Xi_2^2 + (3-\xi)\Xi_3\right] U_1^2(q, e^{i\theta}) - (2-\xi)^2 \Xi_2^2 U_2(q, e^{i\theta})\right\}}. \tag{2.11}$$

Next, if we subtract (2.7) from (2.5), we obtain

$$2(3-\xi)\Xi_3\left(a_3 - a_2^2\right) = U_1(q, e^{i\theta})\left(m_2 - r_2\right). \tag{2.12}$$

Then, in view of (2.9), the equation (2.12) becomes

$$a_3 = \frac{U_1^2(q, e^{i\theta})\left(m_1^2 + r_1^2\right)}{2\left(2-\xi\right)^2 \Xi_2^2} + \frac{U_1(q, e^{i\theta})\left(m_2 - r_2\right)}{2(3-\xi)\Xi_3}.$$

Notice that from (2.3), we get desired inequality for $|a_3|$.

From (2.11) and (2.12), we find that

$$\begin{aligned} a_3 - \rho a_2^2 &= \frac{(1-\rho)U_1^3(q,e^{i\theta})(m_2+r_2)}{2\left\{\left[(\xi^2-2\xi)\Xi_2^2+(3-\xi)\Xi_3\right]U_1^2(q,e^{i\theta})-(2-\xi)^2\Xi_2^2 U_2(q,e^{i\theta})\right\}} + \frac{U_1(q,e^{i\theta})(m_2-r_2)}{2(3-\xi)\Xi_3} \\ &= \frac{U_1(q,e^{i\theta})}{2}\left[\left(h\left(\rho\right) + \frac{1}{(3-\xi)\Xi_3}\right) m_2 + \left(h\left(\rho\right) - \frac{1}{(3-\xi)\Xi_3}\right) r_2\right], \end{aligned}$$

where

$$h\left(\rho\right)=\frac{U_1^2(q,e^{i\theta})\left(1-\rho\right)}{\left[(\xi^2-2\xi)\Xi_2^2+(3-\xi)\Xi_3\right]U_1^2(q,e^{i\theta})-\left(2-\xi\right)^2\Xi_2^2U_2(q,e^{i\theta})}.$$

Thus, in view of (2.3), we get

$$\left|a_3-\rho a_2^2\right|\leq\begin{cases}\dfrac{\left|U_1(q,e^{i\theta})\right|}{(3-\xi)\Xi_3}, & 0\leq\left|h\left(\rho\right)\right|\leq\frac{1}{(3-\xi)\Xi_3}\\ \\ \left|h\left(\rho\right)\right|\left|U_1(q,e^{i\theta})\right|, & \left|h\left(\rho\right)\right|\geq\frac{1}{(3-\xi)\Xi_3}\end{cases},$$

which evidently completes the proof of Theorem 1. Q.E.D.

Analysis similar to that in the proof of the previous Theorem shows that

Theorem 2.2. Let the function f given by (1.1) be in the class $\mathfrak{K}_{\Sigma}^{a,b,c}(\xi,q,\theta)$. Then

$$|a_2|\leq\frac{\left|e^{i\theta}+qe^{-i\theta}\right|\sqrt{\left|e^{i\theta}+qe^{-i\theta}\right|}}{\sqrt{\left|4(\xi-2)\Xi_2^2\left(2e^{2i\theta}+2q^2e^{-2i\theta}+(\xi+2)q\right)+3(3-\xi)\Xi_3(e^{2i\theta}+q^2e^{-2i\theta}+2q)\right|}},$$

$$|a_3|\leq\frac{\left|e^{i\theta}+qe^{-i\theta}\right|}{3(3-\xi)\Xi_3}+\frac{\left|e^{2i\theta}+q^2e^{-2i\theta}+2q\right|}{4(\xi-2)^2\Xi_2^2}$$

for any real number ρ,

$$\left|a_3-\rho a_2^2\right|\leq$$

$$\begin{cases}\dfrac{\left|e^{i\theta}+qe^{-i\theta}\right|}{3(3-\xi)\Xi_3}, & |\rho-1|\leq Y\\ \\ \dfrac{|1-\rho|\left|e^{2i\theta}+q^2e^{-2i\theta}+2q\right|\left|e^{i\theta}+qe^{-i\theta}\right|}{\left|4(\xi-2)\Xi_2^2\left(2e^{2i\theta}+2q^2e^{-2i\theta}+(\xi+2)q\right)+3(3-\xi)\Xi_3(e^{2i\theta}+q^2e^{-2i\theta}+2q)\right|}, & |\rho-1|\geq Y\end{cases}$$

where $Y=\frac{\left|4(\xi-2)\Xi_2^2\left(2e^{2i\theta}+2q^2e^{-2i\theta}+(\xi+2)q\right)+3(3-\xi)\Xi_3(e^{2i\theta}+q^2e^{-2i\theta}+2q)\right|}{3(3-\xi)\Xi_3\left|e^{2i\theta}+q^2e^{-2i\theta}+2q\right|}$.

3 Conclusion

In this present investigation, we have introduced and studied the coefficient problems associated with the following new subclasses

$$\mathfrak{S}_{\Sigma}^{a,b,c}(\xi,q,\theta)\text{ and }\mathfrak{K}_{\Sigma}^{a,b,c}(\xi,q,\theta)\qquad\left(0\leq\xi\leq1,\ q\in(-1,1],\ \theta\in[-\pi,\pi];\ z,w\in D\right)$$

of the class of normalized bi-univalent functions in the open unit disc D. For functions belonging to these bi-univalent function classes, we have derived Taylor–Maclaurin coefficient inequalities and considered the celebrated Fekete–Szegö problem in Section 2.

The geometric properties of the function classes $\mathfrak{S}_{\Sigma}^{a,b,c}(\xi,q,\theta)$, $\mathfrak{K}_{\Sigma}^{a,b,c}(\xi,q,\theta)$ vary according to the values assigned to the parameters involved. Nevertheless, some results for the special cases of the parameters involved could be presented as illustrative examples.

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References

[1] A. Akgül and Ş. Altınkaya, *Coefficient estimates associated with a new subclass of bi-univalent functions*, Acta Universitatis Apulensis **52** (2017) 121-128.

[2] Ş. Altınkaya, *Application of quasi-subordination for generalized Sakaguchi type functions*, Journal of Complex Analysis, **2017** (2017) 1-5.

[3] Ş. Altınkaya and S. Yalçın, *Faber polynomial coefficient estimates for bi-univalent functions of complex order based on subordinate conditions Involving of the Jackson (p, q) -derivative operator*, Miskolc Mathematical Notes **18** (2017) 555-572.

[4] D. A. Brannan and J. G. Clunie, *Aspects of contemporary complex analysis*, Proceedings of the NATO Advanced Study Institute Held at University of Durham, New York: Academic Press, (1979).

[5] D. A. Brannan and T. S. Taha, *On some classes of bi-univalent functions*, Studia Universitatis Babeş-Bolyai Mathematica **31** (1986) 70-77.

[6] S. Bulut, *Coefficient estimates for a subclass of analytic bi-univalent functions by means of Faber polynomial expansions*, Palestine Journal of Mathematics, **7** (2018) 53-59.

[7] P. L. Chebyshev, *Complete Collected Works*, Moscov-Leningrad, (1947).

[8] P. L. Duren, *Univalent Functions*, Grundlehren der Mathematischen Wissenschaften, Springer, New York, USA 259, (1983).

[9] E. Deniz, J. M. Jahangiri, S. G. Hamidi and S. S. Kına, *Faber polynomial coefficients for generalized bi-subordinate functions of complex order*, Journal of Mathematical Inequalities **12** (2018) 645–653.

[10] J. Dziok and H. M. Srivastava, *Classes of analytic functions associated with the generalized hypergeometric function*, Applied Mathematics and Computation **103** (1999) 1-13.

[11] J. Dziok and H. M. Srivastava, *Certain subclasses of analytic functions associated with the generalized hypergeometric function*, Integral Transforms and Special Functions. **14** (2003) 7-18.

[12] M. Fekete and G. Szegö, *Eine Bemerkung Uber Ungerade Schlichte Funktionen*, Journal of London Mathematical Society **[s1-8 (2)]** (1933) 85–89.

[13] S. G. Hamidi and J. M. Jahangiri, *Faber polynomial coefficients of bi-subordinate functions*, Comptes Rendus Mathematique **354** (2014) 365-370.

[14] Yu. E. Hohlov, *Hadamard convolutions, hypergeometric functions and linear operators in the class of univalent functions*, Dokl. Akad. Nauk Ukrain. **7** (1984) 25–27.

[15] Yu. E. Hohlov, *Convolution operators that preserve univalent functions*, Ukrainian Mathematical Journal **37** (1985) 220–226.

[16] G. Murugusundaramoorthy and T. Bulboaca, *Estimate for initial MacLaurin coefficients of certain subclasses of bi-univalent functions of complex order associated with the Hohlov operator*, Annales Universitatis Paedagogicae Cracoviensis. Studia Mathematica **17** (2018) 27-36.

[17] E. Netanyahu, *The minimal distance of the image boundary from the origin and the second coefficient of a univalent function in* $|z| < 1$, Archive for Rational Mechanics and Analysis **32** (1969) 100-112.

[18] F. M. Sakar, E. Dogan, *On initial Chebyshev polynomial coefficient problem for certain subclass of bi-univalent functions*, Communications in Mathematics and Applications **11** (2020) 57-64.

[19] H. M. Srivastava, A. K. Mishra and P. Gochhayat, *Certain subclasses of analytic and bi-univalent functions*, Applied Mathematics Letters **23** (2010) 1188-1192.

[20] H. M. Srivastava, Ş. Altınkaya and S. Yalçın, *Hankel determinant for a subclass of bi-univalent functions defined by using a symmetric q-derivative operator*, Filomat **32** (2018) 503-516.

[21] H. M. Srivastava, F. M. Sakar and H. Ö. Güney, *Some general coefficient estimates for a new class of analytic and bi-univalent functions defined by a linear combination*, Filomat **32** (2018) 1313–1322.